

Activities Using a Vectors Applet and Nelson's *Calculus and Vectors* Student Book (SB)

Vectors Applet Link: <http://oame.on.ca/main/files/gr12-2007/MCV4U/3D/3Dplot.htm>

Chapter 6: Introduction to Vectors

In this chapter, students will:

- represent vectors as directed line segments, **Section 6.1**
- recognize a vector as a quantity with both magnitude and direction, **Section 6.1**
- perform mathematical operations on geometric vectors, **Sections 6.2, 6.3**
- determine the properties of the operations performed on vectors, **Section 6.4**
- determine the Cartesian representation of a vector in two- and three-dimensional space, **Sections 6.5, 6.6, 6.7, 6.8**
- perform mathematical operations on algebraic vectors in two- and three-dimensional space, **Sections 6.6, 6.7, 6.8**

Activity		Student Book Section	Student Book Examples	Student Book Questions
1	Graphing a Point on an Axis	6.5 Vectors in R^2 and R^3		6a, 8
2	Relating Points and Vectors	6.5 Vectors in R^2 and R^3		6b, 7a
3	Representing Opposite Vectors	6.5 Vectors in R^2 and R^3		
4	Representing the Sum and Difference of Vectors in R^2	6.6 Operations with Algebraic Vectors in R^2	Example 2	
5	Representing the Sum and Difference of Vectors in R^2	6.7 Operations with Vectors in R^3		

Activity 1: Graphing a Point on an Axis

Section 6.5—Vectors in R^2 and R^3

To graph a point on the positive x -axis:

- Click: Reset All
- Select: Points
Select: Pt./Vector 1
Select: point
- Select the 0 in $(0, 0, 0)$ which you predict would change the coordinates to name a point on the positive x -axis.
Click on the right or left arrow, or move the scroll bar, to show your prediction.

Observe and Experiment

- How does the diagram show whether the point is on the positive x -axis?
How do the coordinates show this?
- Drag the axes. What do you notice about the position of the point?
- Justify and assess your prediction about the coordinates of a point on the positive x -axis.

To graph a point on the negative x -axis:

- Click: Reset All
- Select: Pt./Vector 2
Select: point
- Select the 0 in $(0, 0, 0)$ which you predict would change the coordinates to name a point on the negative x -axis.
Click on an arrow, or move the scroll bar, to show your prediction.
- Then, click on an arrow, or move the scroll bar, until the point is on the positive x -axis.

Observe and Experiment

- Drag the axes. What do you notice about the position of the points?
- How are the coordinates of a point on the positive x -axis the same as the coordinates of a point on the negative x -axis? How are they different?
Use your answers to describe the coordinates of a point on the x -axis.
- Follow a similar procedure to graph a point on the positive y -axis, on the negative y -axis, on the positive z -axis, then on the negative z -axis.
Explain how you adapted the procedure for each axis.
- Can you move a point off an axis by dragging the axes?
Can you move a point to a different axis by dragging the axes?
How do your experiments show this?

Apply and Adapt

- Use or adapt the procedure to represent or check your answers for *Section 6.5* questions 6a and 8. Choose one point in these questions and explain how you adapted the procedure.

Activity 2: Relating Points and Vectors

Section 6.5—Vectors in R^2 and R^3

To show that \overrightarrow{OP} has its head at point $P(a, b, c)$ and its tail at the origin $O(0, 0, 0)$:

Let $P = (5, -2, 6)$

- Click: Reset All

To graph point $P(5, -2, 6)$:

- Select: Points
Select: Pt./Vector 1
Select: point
- Select the first 0 in $(0, 0, 0)$.
Click on the right or left arrow, or move the scroll bar, to show 5.
Select the next 0 in $(5, 0, 0)$. Click on an arrow, or move the scroll bar, to show -2 .
Select the next 0 in $(5, -2, 0)$. Change it to show 6.

To graph the vector $\overrightarrow{OP} = (5, -2, 6)$:

- Select: Pt./Vector 2
Select: vector
- Change $(0, 0, 0)$ to show $(5, -2, 6)$.

Observe and Experiment:

- How are the diagrams for point $P(5, -2, 6)$ and vector $\overrightarrow{OP} = (5, -2, 6)$ the same? How are they different? Drag the axes. Does the diagram always show the same relationship between the point and the vector?
- How does the diagram show that \overrightarrow{OP} has its head at point $P(5, -2, 6)$ and its tail at the origin $O(0, 0, 0)$?
- Repeat the procedure for another point and vector with its tail at the origin and its head at the point.
- Make a conjecture about vectors that you can check on the diagram. Check your conjecture.

Apply and Adapt

- Use the procedure to represent or check your answers for *Section 6.5* questions 6b and 7a.

Activity 3: Representing Opposite Vectors

Section 6.5—Vectors in R^2 and R^3

To represent opposite vectors along an axis:

- Click: Reset All

To graph the vector $\vec{OA} = (14, 0, 0)$:

- Select: Points
Select: Pt./Vector 1
Select: vector
Select the first 0 in $(0, 0, 0)$. Click on an arrow, or move the scroll bar, to show 14.

To graph the vector opposite \vec{OA} :

- Select: Points
Select: Pt./Vector 2
Select: vector
- Predict how to change $(0, 0, 0)$ to name the vector opposite \vec{OA} .
Change the values in $(0, 0, 0)$ to show your prediction.

Observe and Experiment:

- Consider the directions of the vectors on the diagram.
Count along the axis to determine the magnitude of each vector.
Drag the axes if necessary to make the marks on the axes visible.
Are the vectors opposites? How does the diagram show this?
How do the coordinates show this?
- Describe how you made your prediction.
Explain whether you agree with your prediction and with your strategy for predicting.
- Use a similar procedure to graph two opposite vectors along a different axis.
What does the diagram show about whether these vectors are opposites?
Make a conjecture about the coordinates of opposite vectors along an axis.
Check your conjecture. Explain your strategy for checking it.

To represent opposite vectors that are not along an axis:

- Click: Reset All

To graph the vector $\vec{OA} = (-10, 3, 5)$:

- Select: Points
Select: Pt./Vector 1
Select: vector
- Select the first 0 in $(0, 0, 0)$. Click on an arrow, or move the scroll bar, to show -10.
Select the next 0 in $(-10, 0, 0)$. Change it to show 3.
Select the next 0 in $(-10, 3, 0)$. Change it to show 5.

To graph the vector opposite \overrightarrow{OA} :

- Select: Points
Select: Pt./Vector 2
Select: vector
- Select the first 0 in (0, 0, 0).
Click on an arrow, or move the scroll bar, to show the opposite of -10 .
Select the next 0. Change it to show the opposite of 3.
Select the next 0. Change it to show the opposite of 5.

Observe and Experiment

- Do the vectors look as though they are opposites? Explain.
- Drag the axes. What happened to the vectors?
- Make a conjecture about the coordinates of opposite vectors. Justify your conjecture.
Graph other pairs of opposite vectors. Explain what they show about your conjecture.

Activity 4: Representing the Sum and Difference of Vectors in R^2

Section 6.6—Operations with Algebraic Vectors in R^2

EXAMPLE 2

Given $\vec{a} = \overrightarrow{OA} = (1, 3)$ and $\vec{b} = \overrightarrow{OB} = (4, -2)$, determine the components of $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ and illustrate each of these vectors on the diagram.

To represent the sum:

Solution

The sum, \overrightarrow{OC} , is $(1, 3) + (4, -2) = (1 + 4, 3 + (-2)) = (5, 1)$.

- Click: Reset All

To graph $\overrightarrow{OA} = (1, 3)$:

- Select: Points
Select: Pt./Vector 1
Select: vector
Select the first 0 in (0, 0, 0).
Click on an arrow, or move the scroll bar, to show 1.
- Select the next 0 in (1, 0, 0).
Click on an arrow, or move the scroll bar, to show 3. [Leave the last 0 as it is.]

To graph $\overrightarrow{OB} = (4, -2)$:

- Select: Points
Select: Pt./Vector 2
Select: vector
Select the first 0 in (0, 0, 0).
Click on an arrow, or move the scroll bar, to show 4.
Select the next 0 in (4, 0, 0). Change it to show -2 .
[Leave the third 0 as 0.]

To graph $\overrightarrow{OC} = (5, 1)$:

- Select: Points
Select: Pt./Vector 3
Select: vector
Change (0, 0, 0) to show (5, 1, 0).

Observe and Experiment:

- What does \overrightarrow{OC} represent? How does the diagram show this?
- Drag the axes. What do you notice about the vectors?

Solution

The difference, \overrightarrow{OD} , is $(1, 3) - (4, -2) = (1 - 4, 3 + 2) = (-3, 5)$.

To graph \overrightarrow{OD} $(-3, 5)$:

- Select: Points
Select: Pt./Vector 3
Click: Clear 3
Select: vector
- Enter the coordinates to show $(-3, 5, 0)$.

Observe and Experiment:

- Drag the axes. What do you notice about the vectors?
- What does \overrightarrow{OD} represent? How does the diagram show this?

Activity 5: Representing the Sum and Difference of Vectors in R^3

Section 6.7—Operations with Vectors in R^3

Given $\vec{OA} = (-1, 2, 4)$ and $\vec{OB} = (-2, 0, -1)$, show $\vec{OA} + \vec{OB}$ and $\vec{OA} - \vec{OB}$.

Solution

The sum for $\vec{OA} + \vec{OB}$ is $(-1, 2, 4) + (-2, 0, -1) = (-1 + (-2), 2 + 0, 4 + (-1)) = (-3, 2, 3)$.

Click: Reset All

To graph $\vec{OA} = (-1, 2, 4)$:

- Select: Points
Select: Pt./Vector 1
Select: vector
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show -1 .
Select the next 0 in $(-1, 0, 0)$. Change it to show 2.
Change the next 0 in $(-1, 2, 0)$ to show 4.

To graph $\vec{OB} = (-2, 0, -1)$:

- Select: Points
Select: Pt./Vector 2
Select: vector
- Select the first 0 in $(0, 0, 0)$. Change it to show -2 .
Select the last 0 in $(-2, 0, 0)$. Change it to show -1 .

To graph the sum for $\vec{OA} + \vec{OB}$, $(-3, 2, 3)$:

- Select: Points
Select: Pt./Vector 3
Select: vector
- Enter the coordinates to show the sum $(-3, 2, 3)$.

Observe and Experiment

- How does the diagram show the vectors that are added?
How does it show the sum?
- Drag the axes. What does not change? Why should this not change?

Solution

The difference for $\vec{OA} - \vec{OB}$ is $(-1, 2, 4) - (-2, 0, -1) = (-1 - (-2), 2 - 0, 4 - (-1)) = (1, 2, 5)$.

To graph the difference for $\vec{OA} - \vec{OB}$, $(1, 2, 5)$.

- Select: Points
Select: Pt./Vector 3
Click: Clear 3
Select: vector
- Enter the coordinates to show the difference $(1, 2, 5)$.

Observe and Experiment

- How does the diagram show the vectors that are subtracted?
How does it show the difference?
- Drag the axes. What does not change? Why should this not change?
- How efficient do you think the diagram is for representing subtraction with vectors?
Justify your answer.

Chapter 7: Applications of Vectors

In this chapter, students will:

- use vectors to model and solve problems arising from real-world applications involving velocity and force, **Sections 7.1, 7.2**
- perform the operation of the dot product on two vectors, **Sections 7.3, 7.4**
- determine properties of the dot product, **Sections 7.3, 7.4**
- determine the scalar and vector projections of a vector, **Section 7.5**
- perform the operations of cross-products on two algebraic vectors in three-dimensional space, **Section 7.6**
- determine properties of the cross product, **Section 7.6**
- solve problems involving the dot product and cross product, **Section 7.7**

Activity		Student Book Section	Student Book Examples	Student Book Questions
6	Representing Cross Products	The Cross Product of Two Vectors	2	

Activity 6: Representing Cross Products

Section 7.6—The Cross Product of Two Vectors

EXAMPLE 2

If $\vec{p} = (-1, 3, 2)$ and $\vec{q} = (2, -5, 6)$, calculate $\vec{p} \times \vec{q}$ and $\vec{q} \times \vec{p}$.

Solution

The solution is $\vec{p} \times \vec{q} = (28, 10, -1)$, and $\vec{q} \times \vec{p} = (-28, -10, 1)$.

- Click: Reset All

To graph $\vec{p} \times \vec{q} = (28, 10, -1)$:

- Select: Points
Select: Pt. Vector 1
Select: vector
- Select the first 0 in (0, 0, 0).
Click on an arrow, or move the scroll bar, to show 28.
Select the next 0 in (28, 0, 0). Change it to show 10.
Select the next 0 in (28, 10, 0). Change it to show -1.

To graph $\vec{q} \times \vec{p} = (-28, -10, 1)$:

- Select: Points
Select: Pt. Vector 2
Select: vector
- Change (0, 0, 0) to show (-28, -10, 1).

Observe and Experiment

- Drag the axes and vectors. How does the diagram show that $\vec{p} \times \vec{q}$ and $\vec{q} \times \vec{p}$ are opposite vectors?
- Try this with other cross products. Make a conjecture about cross products. Explain how your experiment supports your conjecture.

Chapter 8: Equations of Lines and Planes

In this chapter, students will:

- determine the vector and parametric equations of a line in two-space, **Section 8.1**
- make connections between Cartesian, vector, and parametric equations of a line in two-space, **Section 8.2**
- determine the vector, parametric, and symmetric equations of a line in three-space, **Section 8.3**
- determine the vector, parametric, and Cartesian equations of a plane, **Sections 8.4, 8.5**
- determine some geometric properties of a plane, **Section 8.5**
- determine the equation of a plane in Cartesian, vector, or parametric form, given another form, **Section 8.5**
- sketch a plane in three-space, **Section 8.6**

Activity		Student Book Section	Student Book Examples	Student Book Questions
7	Representing Points on a Line in R^2	8.1 Vector and Parametric Equations of a Line in R^2	2	2, 4, 5, 6, and student-selected questions
8	Representing Perpendicular Lines in R^2	8.1 Vector and Parametric Equations of a Line in R^2	4	10, and student-selected questions
9	Representing the Equation of the Same Line in Different Forms	8.3 Vector, Parametric, and Symmetric Equations of a Line in R^3	4	3, 4, 6a, 7, and student-selected questions
10	Representing the Equation of the Same Plane in Different Forms and Representing Points	8.4 Vector and Parametric Equations of a Plane	2	7, 9, and student-selected questions
11	Representing a Plane and the Normal to the Plane	8.5 The Cartesian Equation of a Plane	1	1-5, and student-selected questions
12	Representing Parallel and Perpendicular Planes	8.5 The Cartesian Equation of a Plane	4	13-15, and student-created questions
13	Representing Planes in R^3 Whose Equations have One Variable	8.6 Sketching Planes in R^3	1	1-3, 7
14	Representing Direction Vectors for Planes	8.6 Sketching Planes in R^3	7	5b, and student-created questions

Activity 7: Representing Points on a Line in \mathbf{R}^2

Section 8.1—Vector and Parametric Equations of a Line in \mathbf{R}^2

EXAMPLE 2

- Determine the vector and parametric equations of a line passing through point $A(1, 4)$ with direction vector $\vec{m} = (-3, 3)$.
- Sketch the line, and determine the coordinates of four points on the line.
- Is either point $Q(-21, 23)$ or point $R(-29, 34)$ on this lines?

Solution

a. Solutions for the vector equation are $\vec{r} = (1, 4) + t(-3, 3)$, $t \in \mathbf{R}$ and $\vec{r} = (1, 4) + s(-1, 1)$, $s \in \mathbf{R}$ with different parameters. The lines the equations represent are identical. The parametric equation is $x = 1 - s$, $y = 4 + s$, $s \in \mathbf{R}$.

- Click: Reset All

To graph the vector equation $\vec{r} = (1, 4) + t(-3, 3)$, $t \in \mathbf{R}$:

- Select: Lines
Select: Line 1
Select: vector
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show 1.
Select the next 0 in $(1, 0, 0)$. Change it to show 4.
[Leave the last 0 since the line is in \mathbf{R}^2 .]
- Select the first 0 in $t(0, 0, 0)$. Change it to show -3 .
Select the next 0 in $t(-3, 0, 0)$. Change it to show 3.

To graph the vector equation $\vec{r} = (1, 4) + s(-1, 1)$, $s \in \mathbf{R}$:

- Select: Lines
Select: Line 2
Select: vector
- Enter the values to represent $\vec{r} = (1, 4) + s(-1, 1)$, $s \in \mathbf{R}$
[The parameter can be represented by t instead of s .]

To graph the parametric equation $x = 1 - s$, $y = 4 + s$, $s \in \mathbf{R}$.

- Select: Lines
Select: Line 3
Select: parametric
- Select the first 0 in $x = 0 + 0t$. Change it to show 1.
Select the 0 in $0t$ for $x = 1 + 0t$. Change it to show -1 .
[The $+$ sign will remain to show $x = 1 + -1t$.]
- Change $y = 0 + 0t$ to represent the equation $y = 4 + s$.
[Leave $z = 0 + 0t$ since the line is in \mathbf{R}^2 .]

Observe and Experiment

- How do the lines move as the values are entered in the equation?
- How does the diagram show that the equations represent identical lines?
- Do you prefer representing equations on a grid paper or on an online diagram? Justify your choice.

To graph point $A(1, 4)$:

- Select: Points
Select: Pt.Vector 1
Select: point
- Select the first 0 in $(0, 0, 0)$. Change it to show 1.
Select the next 0 in $(1, 0, 0)$. Change it to show 4. [Leave the last 0.]

Observe and Experiment

- Drag the axes and the line. Is it possible to move the point off the line?
- Change a coordinate for the point. Is the point on the line?
Drag the axes and the line.
Is it possible to move a point on or off the line? Explain.

Solution

b. The coordinates of four points on the line are $A(0, 5)$, $B(1, 4)$, $C(2, 3)$, and $D(7 - 2)$.

To graph point $A(0, 5)$:

- Select: Points [Keep the line from part a. in the diagram.]
Select: Pt.Vector 2
Select: Point
- Change the values to show $(0, 5, 0)$ to represent $(0, 5)$.

Observe and Experiment

- How does the diagram show whether the point is on the line?
Does this change when you drag the axes or the lines? Explain.
- Repeat this for points $B(1, 4)$, $C(2, 3)$, and $D(7 - 2)$.

Solution

Point $Q(-21, 23)$ is not on the line. Point $R(-29, 34)$ is on the line.

c. To graph point $Q(-21, 23)$:

- Select: Points [Keep the line from part a. in the diagram.]
Select: Pt.Vector 1
Click on Clear 1.
Select: Point
- Change the values to show $(-21, 23, 0)$.

To graph point $R(-29, 34)$:

- Select: Points [Keep the line from part a. in the diagram.]
Select: Pt.Vector 2
Click on Clear 2.
Select: Point
- Change the values to show $(-29, 34, 0)$.

If the points are not shown, or if the lines do not extend far enough:

- Select: Options
Select: Magnitude
Select: Lines
Click on an arrow, or move the scroll bar, to increase the value.
If necessary, drag the axes.

Observe and Experiment

- How does the diagram show whether a point is on the line?
Does this change when you drag the axes or the lines? Explain.
- What happens if you change a coordinate of a point that is on the line?
- What happens if you change a value for an equation?
- Do you prefer calculating or using the online diagram to determine whether a point is on a given line? Justify your choice.

Apply and Adapt

- Use this procedure to represent or check your answers for *Section 8.1* questions 2, 4, 5, and 6, where the values are integers.
- Choose other questions and apply or adapt the procedure to represent or check your answers.

Activity 8: Representing Perpendicular Lines in R^2

Section 8.1—Vector and Parametric Equations of a Line in R^2

EXAMPLE 4

Determine a vector equation for the line that is perpendicular to $\vec{r} = (4, 1) + s(-3, 2)$, $s \in \mathbf{R}$, and passes through point $P(6, 5)$.

Solution

The vector equation is $\vec{r} = (6, 5) + t(2, 3)$, $t \in \mathbf{R}$.

- Click: Reset All

To graph the vector equation $\vec{r} = (4, 1) + s(-3, 2)$, $s \in \mathbf{R}$:

- Select: Lines
Select: Line 1
Select: vector
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show 4.
Select the next 0 in $(4, 0, 0)$. Change it to show 1.
[Leave the last 0 since the line is in R^2 .]
- Select the first 0 in $t(0, 0, 0)$.
Change it to show -3 .
Select the next 0 in $t(-3, 0, 0)$. Change it to show 2.
[The parameter can be represented by t instead of s .]

To graph the vector equation $\vec{r} = (6, 5) + t(2, 3)$, $t \in \mathbf{R}$:

- Select: Lines
Select: Line 2
Select: vector
- Enter the values to represent $\vec{r} = (6, 5) + t(2, 3)$, $t \in \mathbf{R}$

To graph point $P(6, 5)$:

- Select: Points
Select: Pt.Vector 1
Select: Point
- Change the values to show $(6, 5, 0)$.

Observe and Experiment

- Drag the axes so that the x -axis and the y -axis are perpendicular.
Do the lines $\vec{r} = (4, 1) + s(-3, 2)$, $s \in \mathbf{R}$ and $\vec{r} = (6, 5) + t(2, 3)$, $t \in \mathbf{R}$ look perpendicular?
Explain.
- Drag the axes so that the x -axis and the y -axis are not perpendicular.
Do the lines $\vec{r} = (4, 1) + s(-3, 2)$, $s \in \mathbf{R}$ and $\vec{r} = (6, 5) + t(2, 3)$, $t \in \mathbf{R}$ look perpendicular?
Explain.

Apply and Adapt

- Use or adapt this procedure to represent or check your answers for *Section 8.1* question 10.
- Adapt this procedure for other questions in *Section 8.1*. What did you learn by adapting the procedure?

Activity 9: Representing the Equation of the Same Line in Different Forms

Section 8.3—Vector, Parametric, and Symmetric Equations of a Line in R^3

Using points from EXAMPLE 4

To show that vector, parametric, and symmetric equations represent the same line for a line that passes through points $A(-1, 5, 7)$ and $B(3, -4, 8)$:

The vector equation is $\vec{r} = (-1, 5, 7) + s(-4, 9, -1)$, $s \in \mathbf{R}$.

The parametric equations are $x = -1 - 4t$, $y = 5 + 9t$, $z = 7 - t$, $t \in \mathbf{R}$.

The symmetric equations are $\frac{x+1}{-4} = \frac{y-5}{9} = \frac{z-7}{-1}$.

- Click: Reset All

To graph the vector equation: $\vec{r} = (-1, 5, 7) + s(-4, 9, -1)$, $s \in \mathbf{R}$

- Select: Lines
Select: Line 1
Select: vector
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show -1 .
Select the next 0 in $(-1, 0, 0)$. Change it to show 5.
Select the next 0 in $(-1, 5, 0)$. Change it to show 7.
- Select the first 0 in $t(0, 0, 0)$. [t can be used instead of s .]
Change it to show -4 .
Change the next 0 in $t(-4, 0, 0)$ to show 9.
- Change the next 0 in $t(-4, 9, 0)$ to show -1 .

To graph the parametric equations: $x = -1 - 4t$, $y = 5 + 9t$, $z = 7 - t$, $t \in \mathbf{R}$

- Select: Lines
Select: Line 2
Select: parametric
- Select the first 0 in $x = 0 + 0t$. Change it to show -1 .
Select the 0 in $0t$ for $x = -1 + 0t$. Change it to show -4 . [The $+$ sign will remain.]
- Change $y = 0 + 0t$ to show the equation $y = 5 + 9t$.
Change $z = 0 + 0t$ to show the equation $z = 7 - t$.
[The $+$ sign will remain and the numeral 1 will show for $z = 7 + -1t$.]

To graph the symmetric equations: $\frac{x+1}{-4} = \frac{y-5}{9} = \frac{z-7}{-1}$

- Select: Lines
- Select: Line 3
- Select: parametric
- Select values in the equations.
Use the arrows or scroll bar to enter the values. [The $-$ symbol will remain so that $x - 0$ becomes $x - -1$, representing $x + 1$.]

Observe and Experiment

- How does the position of a line change as the values for the equation are entered?
- How does the position of the line show that the equations represent the same line? How does the colour of the line show that the equations represent the same line?
- Drag the axes and the line. What do you notice? Draw a conclusion based on what you noticed. How confident are you about your conclusion?

Apply and Adapt

- Use this procedure to represent or check your answers for *Section 8.3* questions 3, 4, 6a, and 7.
- Adapt the procedure to represent or check your answers for other questions in *Section 8.3*. Explain your strategy for adapting the procedure.

Activity 10: Representing the Equation of the Same Plane in Different Forms and Representing Points

Section 8.4—Vector and Parametric Equations of a Plane

EXAMPLE 2

- a. Determine a vector equation and the corresponding parametric equation for the plane that contains the points $A(-1, 3, 8)$, $B(-1, 1, 0)$, and $C(4, 1, 1)$.
- b. Do either of the points $P(14, 1, 3)$ or $Q(14, 1, 5)$ lie on this plane?

Solution

a. The vector equation is $\vec{r}(-1, 3, 8) + s(0, 1, 4) + t(5, 0, 1)$, $s, t \in \mathbf{R}$.

The parametric equations are $x = -1 + 5t$, $y = 3 + s$, $z = 8 + 4s + t$, $s, t \in \mathbf{R}$.

- Click: Reset All

To graph the vector equation $\vec{r}(-1, 3, 8) + s(0, 1, 4) + t(5, 0, 1)$, $s, t \in \mathbf{R}$:

- Select: Planes
Select: Plane 1
Select: vector
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show -1 .
Select the next 0 in $(-1, 0, 0)$. Change it to show 3.
Select the next 0 in $(-1, 3, 0)$. Change it to show 8.
- Select the second 0 in $s(0, 0, 0)$. Change it to show 1.
Select the next 0 in $s(0, 1, 0)$. Change it to show 4.
- Enter the values for $t(5, 0, 1)$.

To graph the parametric equations $x = -1 + 5t$, $y = 3 + s$, $z = 8 + 4s + t$:

- Select: Planes
Select: Plane 2
Select: parametric
- Select the first 0 in $x = 0 + 0t + 0s$. Change it to show -1 .
Select the 0 in $0t$ for $x = -1 + 0t + 0s$. Change it to show 5.
- Select the first 0 in $y = 0 + 0t + 0s$. Change it to show 3.
Select the 0 in $0s$ for $y = 0 + 0t + 0s$. Change it to show 1.
- Enter the values to represent $z = 8 + 4s + t$.

Observe and Experiment

- How do the planes change position as values are entered?
How do the final positions show that the equations represent the same plane?
How do the colours show that the equations represent the same plane?
Do the colours remain the same when you drag the axes or the planes?
- On the screen, the values for t and s are in a different order than in your text.
Predict what would happen if you reversed the values for s and t in all equations.
Check your prediction.
Predict what would happen if you reversed the values for s and t in some equations.
Check your prediction.

Apply and Adapt

- Use this procedure to represent or check answers for *Section 8.4* question 6 b.
- Adapt the procedure to represent or check answers for *Section 8.4* questions 10 and 11. Explain how you adapted the procedure when a point and a line are given instead of three points.

Solution

b. Point $P(14, 1, 3)$ is on the plane. Point $Q(14, 1, 5)$ is not on the plane.

- Keep the plane from part a on the diagram.

To graph point $P(14, 1, 3)$:

- Select: Points.
Select: Pt./Vector 1
Select: point
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show 14.
Select the next 0 in $(14, 0, 0)$. Change it to show 1.
Select the next 0 in $(14, 1, 0)$. Change it to show 3.

Observe and Experiment

- How does the diagram show whether $P(14, 1, 3)$ is on the plane?
- Drag objects in the diagram. Does the diagram still show the same relationship? Do your observations make sense? Explain.

To graph point $Q(14, 1, 5)$:

- Select: Points.
Select: Pt./Vector 2
Select: point
- Enter the coordinates for $Q(14, 1, 5)$.

Observe and Experiment

- How does the diagram show whether point $Q(14, 1, 5)$ is on the plane?
- Drag objects in the diagram. Does the diagram still show the same relationship? How clear do you think the diagram is for showing a relationship between a point and a plane?

Apply and Adapt

- Use this procedure to represent or check answers for *Section 8.4* questions 7 and 9.
- Adapt the procedure to represent or check answers for other questions in *Section 8.4*. Explain how you chose the answers to check.

Activity 11: Representing a Plane and the Normal to the Plane

Section 8.5—The Cartesian Equation of a Plane

EXAMPLE 1

The point $A(1, 2, 2)$ is a point on the plane with normal $\vec{n} = (-1, 2, 6)$.

Determine the Cartesian equation of this plane.

Solution

The Cartesian equation for the plane is $x - 2y - 6z + 15 = 0$.

- Click: Reset All

To graph the plane $x - 2y - 6z + 15 = 0$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$.
Click on an arrow, or move the scroll bar, to show 1. [The numeral 1 will be displayed.]
Select the 0 in $0y$. Change it to show -2 . [The $+$ symbol will remain.]
Select the 0 for $0z$. Change it to show -6 .
- Select the 0 on the right side of the equal sign. Change it to show -15 .
The value is -15 since $x - 2y - 6z + 15 = 0$ results in $x - 2y - 6z = 15$.]

To graph point $A(1, 2, 2)$:

- Select: Points
Select: Pt./Vector 1
Select: point
- Enter values to show $(1, 2, 2)$

To graph normal $\vec{n} = (-1, 2, 6)$:

- Select: Points
Select: Pt./Vector 2
Select: vector
- Enter values to show $(-1, 2, 6)$

Observe and Experiment

- Drag objects in the diagram so that you see only the edge of the plane.
Describe the relationship between the plane and the vector.
- Does it matter which form of an equation you use? Explain.

Apply and Adapt

- Use this procedure or a similar procedure to represent or check answers for *Section 8.5* questions 1 to 5.
- Use or adapt the procedure to represent or check other answers for *Section 8.5*.

Activity 12: Representing Parallel and Perpendicular Planes

Section 8.5—The Cartesian Equation of a Plane

EXAMPLE 4

- a. Show that the planes $\pi_1 : 2x - 3y + z - 1 = 0$ and $\pi_2 : 4x - 3y - 17z = 0$ are perpendicular.
- b. Show that the planes $\pi_3 : 2x - 3y + 2z - 1 = 0$ and $\pi_4 : 2x - 3y + 2z - 3 = 0$ are parallel but not coincident.

a.

- Click: Reset All

To graph the plane $\pi_1 : 2x - 3y + z - 1 = 0$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$.
Click on an arrow, or move the scroll bar, to show 2.
Select the 0 in $0y$. Change it to show -3 . [The + symbol will remain.]
Select the 0 for $0z$. Change it to show 1. [The numeral 1 will show.]
- Select the 0 on the right side of the equal sign. Change it to show 1.

To graph the plane $\pi_2 : 4x - 3y - 17z = 0$:

- Select: Planes
Select: Plane 2
Select: scalar
- Enter the values to represent the equation $4x - 3y - 17z = 0$.

Observe and Experiment

- Drag objects in the diagram. How can you show that the planes are parallel?
- Change values in either equation. Describe how the relationship between the planes changes?

b.

- Click: Reset All

To graph the plane $\pi_3 : 2x - 3y + 2z - 1 = 0$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter the values to represent the equation $2x - 3y + 2z - 1 = 0$.

To graph the plane $\pi_4 : 2x - 3y + 2z - 3 = 0$:

- Select: Planes
Select: Plane 2
Select: scalar
- Enter the values to represent the equation $2x - 3y + 2z - 3 = 0$.

Observe and Experiment

- Drag objects in the diagram. How can you show that the planes are parallel?
- Predict how you could change values in the equations so that the planes are coincident. Check your prediction.

Apply and Adapt

- Use this procedure or a similar procedure to represent or check answers for *Section 8.5* questions 13 to 15.
- Create a question for which you could use this procedure to check the answer. Answer the question. Then use the procedure to check the answer.

Activity 13: Representing Planes in R^3 Whose Equations have One Variable

Section 8.6—Sketching Planes in R^3

EXAMPLE 1

Draw the planes with equations $\pi_1 : x = 5$, $\pi_2 : y = 6$, and $\pi_3 : z = 6$.

- Click: Reset All
To graph the plane $\pi_1 : x = 5$:
- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$. Click on an arrow, or move the scroll bar, to show 1.
Select the 0 on the right side of the equal sign. Change it to show 5.

Observe and Experiment

- Drag objects in the diagram to show that $\pi_1 : x = 5$ is parallel to the yz -plane.
How does the diagram show this?
How does the diagram show that the equation of the plane is $x = 5$, not $x = -5$?

To graph the plane $\pi_2 : y = 6$:

- Select: Planes
Select: Plane 2
Select: scalar
- Select the 0 in $0y$. Click on an arrow, or move the scroll bar, to show 1.
Select the 0 on the right side of the equal sign. Change it to show 6.

Observe and Experiment

- Drag objects in the diagram to show that $\pi_2 : y = 6$ is parallel to the xz -plane.
How does the diagram show this?
How does the diagram show that the equation of the plane is $y = 6$, not $y = -6$?

To graph the plane $\pi_3 : z = 6$:

- Select: Planes
Select: Plane 3
Select: scalar
- Select the 0 in $0z$. Click on an arrow, or move the scroll bar, to show 1.
Select the 0 on the right side of the equal sign. Change it to show 6.

Observe and Experiment

- Drag objects in the diagram to show that $\pi_3 : z = 6$ is parallel to the xy -plane.
How does the diagram show this?
How does the diagram show that the equation of the plane is $z = 6$, not $z = -6$?
- Change the value of the x -intercept for $\pi_1 : x = 5$. How does the diagram change?
Is the plane still parallel to the yz -plane? How does the diagram show this?
Use a similar procedure for $\pi_2 : y = 6$ and $\pi_3 : z = 6$.
- Predict how to change the values of intercepts so that each plane is along an axis.
Check your prediction.

Apply and Adapt

- Use this procedure or a similar procedure to represent or check answers for *Section 8.5* questions 1 to 3, and 7. Explain whether you found the procedure useful for representing the answers.

Activity 14: Representing Direction Vectors for Planes

Section 8.6—Sketching Planes in R^3

EXAMPLE 7

Determine two direction vectors for the planes $\pi_1 : 3x + 4y = 12$ and $\pi_2 : x - y - 5z = 0$.

Solution

Direction vectors for the plane $\pi_1 : 3x + 4y = 12$ are $\vec{m}_1 = (4, -3, 0)$ and $\vec{m}_2 = (0, 0, 1)$.

- Click: Reset All

To graph the plane $\pi_1 : 3x + 4y = 12$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$. Click on an arrow, or move the scroll bar, to show 3.
Select the 0 in $0y$. Change it to show 4.
Select the 0 on the right side of the equal sign. Change it to show 12.

To graph vector $\vec{m}_1 = (4, -3, 0)$:

- Select: Points
Select: Pt.Vector 1
Select: vector
- Change the values to show $(4, -3, 0)$.

To graph vector $\vec{m}_2 = (0, 0, 1)$:

- Select: Points
Select: Pt.Vector 2
Select: vector
- Change the values to show $(0, 0, 1)$

Observe and Experiment

- Drag the plane so that you see only the edge of the plane.
Describe the relationship between the plane and the direction vectors.
How does this relationship fit with the name direction vector?

Solution

Direction vectors for the plane $\pi_2 : x - y - 5z = 0$ are $\vec{m}_1 = (5, 0, 1)$ and $\vec{m}_2 = (1, 1, 0)$.

- Click: Reset All

To graph the plane $\pi_2 : x - y - 5z = 0$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter values to represent the equation $x - y - 5z = 0$.

To graph vector $\vec{m}_1 = (5, 0, 1)$:

- Select: Points
Select: Pt.Vector 1
Select: vector
- Change the values to show (5, 0, 1).

To graph vector $\vec{m}_2 = (1, 1, 0)$:

- Select: Points
Select: Pt.Vector 2
Select: vector
- Change the values to show (1, 1, 0)

Observe and Experiment

- Drag the plane so that you see only the edge of the plane.
Describe the relationship between the plane and the vector.
Explain how well you think the diagram shows the relationship.

Apply and Adapt

- Use this procedure or a similar procedure to represent or check answer for *Section 8.6* question 5b.
- Alter a question in *Section 8.6*, determine the answer, then use the procedure to represent or check the answer.

Chapter 9: Relationships Between Points, Lines, and Planes

In this chapter, students will:

- determine the intersection between a line and a plane and between two lines in three-dimensional space, **Section 9.1**
- algebraically solve systems of equations involving up to three equations in three unknowns, **Section 9.2**
- determine the intersection of two or three planes, **Sections 9.3, 9.4**
- determine the distance from a point to a line in two- and three-dimensional space, **Section 9.5**
- determine the distance from a point to a plane, **Section 9.6**
- solve distance problems relating to lines and planes in three-dimensional space and interpret the result geometrically, **Sections 9.5, 9.6**

Activity		Student Book Section	Student Book Examples	Student Book Questions
15	Representing the Point of Intersection between a Line and a Plane	9.1 The Intersection of a Line with a Plane and the Intersection of Two Lines	1	7a and student-selected questions such as 7b
16	Showing Whether a Line and a Plane Intersect	9.1 The Intersection of a Line with a Plane and the Intersection of Two Lines	2	5b and student-selected questions
17	Representing a Line on a Plane	9.1 The Intersection of a Line with a Plane and the Intersection of Two Lines	3	4 and student-selected questions
18	Representing Solutions for Systems of Equation	9.2 Systems of Equations	3, 4	2, 5, 6, 9, 10, 12, and student-selected questions
19	Representing Parallel, Coincident, or Intersecting Planes, and Lines of Intersection for Two Planes	9.3 The Intersection of Two Planes	Examples 1, 2, 4	6, 7, and student-selected questions
20	Representing Parallel, Coincident, or Intersecting Planes, and Lines of Intersection for Three Planes	9.4 The Intersection of Three Planes	1, 3, 4, 5	1-10 and student-selected questions

Activity 15: Representing the Point of Intersection between a Line and a Plane

Section 9.1—The Intersection of a Line with a Plane and the Intersection of Two Lines

EXAMPLE 1

Determine the point of intersection between the line $L: \vec{r} = (3, 1, 2) + s(1, -4, -8)$, $s \in \mathbf{R}$, and the plane $\pi: 4x + 2y - z - 8 = 0$, if any exist.

Solution

The point of intersection is (2, 5, 10).

- Click: Reset All

To graph the plane $\pi: 4x + 2y - z - 8 = 0$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$.
Click on an arrow, or move the scroll bar, to show 4.
Select the 0 in $0y$. Change it to show 2.
Select the 0 for $0z$. Change it to show -1 .
[The + symbol will remain and the numeral 1 will be displayed to show $+ - 1z$.]
- Select the 0 on the right side of the equal sign. Change it to show 8.
The value is 8 since $4x + 2y - z - 8 = 0$ results in $4x + 2y - z = 8$.]

To graph the line $L: \vec{r} = (3, 1, 2) + s(1, -4, -8)$, $s \in \mathbf{R}$:

- Select: Lines
Select: Line 1
Select the same form as the equation of the line: vector
- Select the first 0 in (0, 0, 0).
Click on an arrow, or move the scroll bar, to show 3.
Select the next 0 in (3, 0, 0). Change it to show 1.
Select the next 0 in (3, 1, 0). Change it to show 2.
- Change $t(0, 0, 0)$ to show $t(1, -4, -8)$. [t can represent the parameter instead of s .]

To graph the point (2, 5, 10):

- Select: Points
Select: Pt./Vector 1
- Select: point
Enter the coordinates for point (2, 5, 10).

Observe and Experiment

- Drag objects in the diagram. Is the point of intersection always on the line? Is the point of intersection always on the plane?

What does this show about whether point $(2, 5, 10)$ is the point of intersection between the line and the plane? Can you use the diagram to make a general statement about the point of intersection between a plane and a line? Justify your answer.

Apply and Adapt

- Use this procedure to represent or check your answer for *Section 9.1* question 7a.
- Use a similar procedure to represent or check your answers for other questions in *Section 9.1* such as 7b. Explain how and why you adapted the procedure.

Activity 16: Showing Whether a Line and a Plane Intersect

Section 9.1—The Intersection of a Line with a Plane and the Intersection of Two Lines

EXAMPLE 2

Determine the point of intersection between the line $L : x = 2 + t, y = 2 + 2t, z = 9 + 8t, t \in \mathbf{R}$ and the plane $\pi : 2x - 5y + z - 6 = 0$, if any exist.

Solution

The planes are parallel and do not intersect.

- Click: Reset All

To graph the plane $\pi : 2x - 5y + z - 6 = 0$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$.
Click on an arrow, or move the scroll bar, to show 2.
Select the 0 in $0y$. Change it to show -5 . [The $+$ symbol will remain to show $+ - 5y$.]
Select the 0 in $0z$. Change it to show 1. [The numeral 1 will be displayed to show $+ 1z$.]
- Select the 0 on the right side of the equal sign. Change it to show 6.
[The value is 6 since $2x - 5y + z - 6 = 0$ results in $2x - 5y + z = 6$.]

To graph the line $L : x = 2 + t, y = 2 + 2t, z = 9 + 8t, t \in \mathbf{R}$:

- Select: Lines
- Select: Line 1
Select: parametric
- Select the first 0 in $x = 0 + 0t$.
Click on an arrow, or move the scroll bar, to show 2.
Select the 0 in $0t$ for $x = 2 + 0t$.
Click on an arrow, or move the scroll bar, to show 1. [The numeral 1 will be displayed to show $+ 1t$.]
- Enter the values to show $y = 2 + 2t$. Enter the values to show $z = 9 + 8t$.

Observe and Experiment

- Can you drag the axes, the plane, or the line so that the line and the plane are not touching? How does this show whether the line and the plane intersect?
- Make a conjecture about how a diagram would show whether a line and a plane intersect. Justify your conjecture. Then, check your conjecture. Explain how you decided to check the conjecture.

Apply and Adapt

- Use this procedure to represent or check your answer for *Section 9.1* question 5b.
- Use a similar procedure to represent or check your answers for other questions in *Section 9.1* such as 5a. Explain how you adjusted the procedure and why this is appropriate for the questions.

Activity 17: Representing a Line on a Plane

Section 9.1—The Intersection of a Line with a Plane and the Intersection of Two Lines

EXAMPLE 3

Determine points of intersection of the line $L : \vec{r} = (3, -2, 1) + s(14, -5, -3), s \in \mathbf{R}$ and the plane $x + y + 3z - 4 = 0$, if any exist.

Solution

There are an infinite number of solutions since the line lies on the plane.

- Click: Reset All

To graph the line $L : \vec{r} = (3, -2, 1) + s(14, -5, -3), s \in \mathbf{R}$:

- Select: Lines
- Select: Line 1
Select: vector
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show 3.
Click on the next 0 in $(3, 0, 0)$. Change it to show -2 .
Click on the next 0 in $(3, -2, 0)$. Change it to show 1.

To graph the plane $x + y + 3z - 4 = 0$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$.
Click on an arrow, or move the scroll bar, to show 1.
Select the 0 in $0y$. Change it to show 1.
Select the 0 in $0z$. Change it to show 3.
- Select the 0 on the right side of the equal sign. Change it to show 4.
[The value is 4 since $x + y + 3z - 4 = 0$ results in $x + y + 3z = 4$.]

Observe and Experiment

- Can you drag the axes, the plane, or the line so that the line is not on the plane?
How does this show whether the line and the plane have an infinite number of solutions?
- Make a conjecture about how a diagram would show whether every point on a line is on the plane. Justify your conjecture. Explain whether you think this conjecture is useful.

Apply and Adapt

- Use this procedure to represent or check your answers for *Section 9.1* question 4, selecting the appropriate form of the equation.
- Use a similar procedure to represent or check your answers for other questions in *Section 9.1*.

Activity 18: Representing Solutions for Systems of Equations

Section 9.2—Systems of Equations

EXAMPLE 3

Determine whether $x = -3$, $y = 5$, and $z = 6$ is a solution to the following system:

1. $2x + 3y - 5z = -21$
2. $x - 6y + 6z = 8$

Solution

They are not a solution to this system.

- Click: Reset All

To graph the plane $2x + 3y - 5z = -21$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$. Click on an arrow, or move the scroll bar, to show 2.
Select the 0 in $0y$. Change it to show 3.
Select the 0 in $0z$. Change it to show -5 . [The $+$ symbol will remain to show $+ - 5z$.]
Select the 0 on the right side of the equal sign. Change it to show -21 .

To graph the plane $x - 6y + 6z = 8$:

- Select: Planes
Select: Plane 2
Select: scalar
- Enter the values to represent the equation $x - 6y + 6z = 8$.

To graph $x = -3$, $y = 5$, and $z = 6$:

- Select: Points
Select: Pt.Vector 1
Select: point
- Select the first 0 in $(0, 0, 0)$.
Click on an arrow, or move the scroll bar, to show -3 .
Select the next 0 in $(-3, 0, 0)$. Change it to 5.
Select the next 0 in $(-3, 5, 0)$. Change it to 6.

Observe and Experiment

- Drag the axes or planes. How does the diagram show that the point is not a solution to the system?
- What method do you prefer for determining whether a point is a solution to a system? Justify your choice.

EXAMPLE 4

Solve the following system of equations for x , y , and z using elementary operations:

1. $x - y + z = 1$
2. $2x + y - z = 11$
3. $3x + y + 2z = 12$

Solution

The solution to this system is $(4, 2, -1)$.

- Click: Reset All

To graph the plane $x - y + z = 1$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter the values to represent the equation $x - y + z = 1$.

To graph the planes $2x + y - z = 11$ and $3x + y + 2z = 12$:

- Select: Planes
- Enter $2x + y - z = 11$ as the scalar equation for Plane 2.
- Enter $3x + y + 2z = 12$ as scalar equation for Plane 3.

To graph $(4, 2, -1)$:

- Select: Points
Select: Pt.Vector 1
Select: point
- Select the first 0 in $(0, 0, 0)$. Change it to 4.
Select the next 0 in $(4, 0, 0)$. Change it to 2.
Select the next 0 in $(4, 2, 0)$. Change it to -1 .

Observe and Experiment

- Drag the axes or planes. Is it possible to move the point off the three planes?
How does this show that the point is a solution to the system?
- Change a coordinate of the point. Drag the axes or planes.
Is it possible to drag the point off the three planes?
What does this show about whether the point is a solution to the system?

Apply and Adapt

- Use this procedure to represent or check your answers for *Section 9.2* questions 2 and 12, where the values are integers.
- Adapt this procedure for questions in R^2 for questions 5 and 6.
Explain how the procedure is the same for R^2 and R^3 .
- Adapt this procedure for other questions in the section such as questions 9 and 10.
How efficient do you consider this procedure for checking answers? Justify your answer.

Activity 19: Representing Parallel, Coincident, or Intersecting Planes, and Lines of Intersection for Two Planes

Section 9.3—The Intersection of Two Planes

EXAMPLE 1

Determine the solution to the system of equations $x - y + z = 4$ and $x - y + z = 5$. Discuss how these planes are related to each other.

Solution

The planes are parallel and non-coincident planes.

- Click: Reset All

To graph the plane $x - y + z = 4$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$. Change it to show 1.
[The numeral 1 will be displayed to show $1x$.]
Select the 0 in $0y$. Change it to show -1 . [The $+$ symbol will remain to show $+ - 1y$.]
Select the 0 in $0z$. Change it to show 1.
Select the 0 on the right side of the equal sign. Change it to show 4.

To graph the plane $x - y + z = 5$:

- Select: Planes
Select: Plane 2
Select: scalar
- Enter the values to show $x - y + z = 5$.

Observe and Experiment

- Drag the axes or planes. How does the diagram show whether the planes are parallel?
- Change the value of 5 in $x - y + z = 5$. Do the planes remain parallel? Explain.
Describe how the diagram shows this. What happens when you increase the value of 5?
What happens when you decrease the value of 5?
- Change other values in the equation. Describe what happens to the diagram.
- Plan a way you might experiment with this diagram. Conduct your experiment.

EXAMPLE 2

Determine the solution to the following system of equations:

1. $x + 2y - 3z = -1$

2. $4x + 8y - 12z = -4$

Solution

There are an infinite number of solutions.

- Click: Reset All

To graph the plane $x + 2y - 3z = -1$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter the values to show $x + 2y - 3z = -1$.

To graph the plane $4x + 8y - 12z = -4$:

- Select: Planes
Select: Plane 2
Select: scalar
- Enter the values to show $4x + 8y - 12z = -4$.

Observe and Experiment

- How does the diagram show that the planes coincide? Drag the axes or planes. How does the diagram remain the same? Explain why this happens.
- Change the value of the constant in either equation or both equations. Do the planes still coincide? Describe how the diagram shows this. Explain why this happens.
- Change the value of a coefficient in either equation. Do the planes still coincide? Describe how the diagram shows this. Explain why this happens.

EXAMPLE 4

Determine the solution to the following system of equations:

1. $2x - y + 3z = -2$

2. $x - 3z = 1$

Solution

The solution is $x = 3s + 1$, $y = 9s + 4$, $z = s$, $s \in \mathbf{R}$.

- Click: Reset All

To graph the plane $2x - y + 3z = -2$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter the values to show $2x - y + 3z = -2$.

To graph the plane $x - 3z = 1$:

- Select: Planes
Select: Plane 2
Select: scalar
- Enter the values to show $x - 3z = 1$.
[Leave the 0 in $0y$ since the equation $x - 3z = 1$ shows $0y$.]

To graph $x = 3s + 1$, $y = 9s + 4$, $z = s$, $s \in \mathbf{R}$:

- Select: Lines
Select: Line 1
Select: parametric
- Select the first 0 in $x = 0 + 0t$. Change it to show 1.
Select the 0 in $0t$ for $x = 1 + 0t$. Change it to show 3.
- Change $y = 0 + 0t$ to show $y = 4 + 9t$ to represent the equation $y = 9s + 4$.
Change $z = 0 + 0t$ to represent the equation $z = s$.

Observe and Experiment

- Drag the axes or planes. How does the diagram remain the same?
Explain why this happens.
- Change the values in the parametric equation of the solution.
Describe how the diagram changes. Explain why this happens.
- Can you use this applet when the values are not integers? Explain.

Apply and Adapt

- Use this procedure to represent or check your answers for *Section 9.3* questions 6 and 7, where the values are integers.
- Use a similar procedure to represent or check your answers for other questions in *Section 9.3*. Explain how chose these questions.

Activity 20: Representing Parallel, Coincident, or Intersecting Planes, and Lines of Intersection for Three Planes

Section 9.4—The Intersection of Three Planes

EXAMPLE 1

Determine the intersection of three planes with the equations $x - y + z = -2$, $2x - y - 2z = -9$, and $3x + y - z = -2$.

Solution

The solution for the point where the planes intersect is $(-1, 3, 2)$.

- Click: Reset All

To graph the plane $x - y + z = -2$:

- Select: Planes
Select: Plane 1
Select: scalar
- Select the 0 in $0x$. Click on an arrow, or move the scroll bar, to show 1.
Select the 0 in $0y$. Change it to show -1 . [The $+$ symbol will remain to show $+ - 1y$.]
Select the 0 in $0z$. Change it to show 1.
Select the 0 on the right side of the equal sign. Change it to show -2 .

To graph the plane $2x - y - 2z = -9$:

- Select: Planes
Select: Plane 2
Select: scalar
- Enter the values to show $2x - y - 2z = -9$.

To graph the plane $3x + y - z = -2$:

- Select: Planes
Select: Plane 3
Select: scalar
- Enter the values to show $3x + y - z = -2$.

To graph the point $(-1, 3, 2)$:

- Select: Points
Select: Pt./Vector 1
Select: point
- Select the first 0 in $(0, 0, 0)$. Change it to show -1 .
Change the other 0s to show $(-1, 3, 2)$.

Observe and Experiment

- Drag the axes or planes. Is the point always on all three planes?
Explain why this happens.
- Change the values for the point.
Drag the axes or planes. Is the point always on all three planes?
Explain why this happens.

EXAMPLE 3

Determine the solution to the following system of equations:

1. $2x + y + z = 1$
2. $4x - y - z = 5$
3. $8x - 2y - 2z = 10$

Solution

The solution to the system is $x = 1$, $y = s$, $z = -s - 1$, $s \in \mathbf{R}$.

- Click: Reset All

To graph the planes $2x + y + z = 1$, $4x - y - z = 5$, and $8x - 2y - 2z = 10$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter the values to show $2x + y + z = 1$.
- Enter the values to show $4x - y - z = 5$ as Plane 2 and $8x - 2y - 2z = 10$ as Plane 3.

To graph $x = 1$, $y = s$, $z = -s - 1$, $s \in \mathbf{R}$:

- Select: Lines
Select: Line 1
Select: parametric
- Change $x = 0 + 0t$ to show $x = 1 + 0t$ to represent the equation $x = 1$.
Change $y = 0 + 0t$ to show $y = 0 + 1t$ to represent the equation $y = s$.
Change $z = 0 + 0t$ to show $z = -1 - 1t$ to represent the equation $z = -s - 1$.

Observe and Experiment

- Make a conjecture about the result of dragging the axes or planes. Justify your conjecture.
Check your conjecture.

EXAMPLE 4

Determine the solution to the following system of equations:

1. $x - y + z = 1$
2. $x + y + 2z = 2$
3. $x - 5y - z = 1$

Solution

The system is inconsistent. There is no solution.

- Click: Reset All

To graph the planes $x - y + z = 1$, $x + y + 2z = 2$, and $x - 5y - z = 1$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter the values to show $x - y + z = 1$.
- Enter the values to show $x + y + 2z = 2$ as Plane 2 and $x - 5y - z = 1$ as Plane 3.

Observe and Experiment

- Drag the axes or planes. How does the diagram show that the system is inconsistent?
- Is it possible to move the planes so that there is a solution? Explain.
Is using the diagram an efficient strategy? Explain.

EXAMPLE 5

Solve the following system of equations:

1. $x + y + z = 5$
2. $x + y + z = 4$
3. $x + y + z = 5$

Solution

The system is inconsistent. There is no solution.

The equations represent non-consistent parallel planes.

- Click: Reset All

To graph the planes $x + y + z = 5$, $x + y + z = 4$ and $x + y + z = 5$:

- Select: Planes
Select: Plane 1
Select: scalar
- Enter the values to show $x + y + z = 5$.
- Enter the values to show $x + y + z = 4$ as Plane 2 and $x + y + z = 5$ as Plane 3.

Observe and Experiment

- Drag the axes or planes. How does the diagram show that the system is inconsistent?
How does the diagram show that the planes are parallel?
How does the diagram show which equations represent the same plane?
- Make a prediction about the equation of a different plane that is parallel to these planes.
Check your prediction.

Apply and Adapt

- Use this procedure to represent or check your answers for *Section 9.4* questions 1 to 10, where the values are integers.
- Use a similar procedure to represent or check your answers for other questions in *Section 9.4*. Explain how you chose these questions.