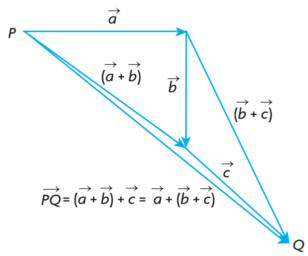
24.

25. **a.** $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$
b. $\sqrt{|\vec{a}|^2 + |\vec{b}|^2}$
c. $\sqrt{4|\vec{a}|^2 + 9|\vec{b}|^2}$

26. **Case 1** If $\vec{b}$ and $\vec{c}$ are collinear, then $2\vec{b} + 4\vec{c}$ is also collinear with both $\vec{b}$ and $\vec{c}$. But $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, so $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.
Case 2 If $\vec{b}$ and $\vec{c}$ are not collinear, then by spanning sets, $\vec{b}$ and $\vec{c}$ span a plane in R^3, and $2\vec{b} + 4\vec{c}$ is in that plane. If $\vec{a}$ is perpendicular to $\vec{b}$ and $\vec{c}$, then it is perpendicular to the plane and all vectors in the plane. So, $\vec{a}$ is perpendicular to $2\vec{b} + 4\vec{c}$.

Chapter 6 Test, p. 348

1. Let P be the tail of $\vec{a}$ and let Q be the head of $\vec{c}$. The vector sums $[\vec{a} + (\vec{b} + \vec{c})]$ and $[(\vec{a} + \vec{b}) + \vec{c}]$ can be depicted as in the diagram below, using the triangle law of addition. We see that $\vec{PQ} = \vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$. This is the associative property for vector addition.



2. **a.** (8, 4, 8)
b. 12
c. $\left(-\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$

3. $\sqrt{19}$

4. **a.** $\vec{x} = 2\vec{b} - 3\vec{a}, \vec{y} = 3\vec{b} - 5\vec{a}$
b. $a = 1, b = 5, c = -11$

5. **a.** $\vec{a}$ and $\vec{b}$ span R^2, because any vector (x, y) in R^2 can be written as a linear combination of $\vec{a}$ and $\vec{b}$. These two vectors are not multiples of each other.
b. $p = -2, q = 3$

6. **a.** $(1, 12, -29) = -2(3, 1, 4) + 7(1, 2, -3)$
b. $\vec{r}$ cannot be written as a linear combination of $\vec{p}$ and $\vec{q}$. In other words, $\vec{r}$ does not lie in the plane determined by $\vec{p}$ and $\vec{q}$.

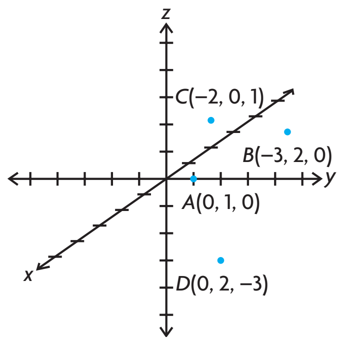
7. $\sqrt{13}, \theta \doteq 3.61$; 73.9° from $\vec{x}$ toward $\vec{y}$

8. $\vec{DE} = \vec{CE} - \vec{CD}$
$\vec{DE} = \vec{b} - \vec{a}$
Also,
$\vec{BA} = \vec{CA} - \vec{CB}$
$\vec{BA} = 2\vec{b} - 2\vec{a}$
Thus,
$\vec{DE} = \frac{1}{2}\vec{BA}$

Chapter 7

Review of Prerequisite Skills, p. 350

1. $v \doteq 806$ km/h N 7.1° E
2. 15.93 units W 32.2° N
3.



4. **a.** $(3, -2, 7); l \doteq 7.87$
b. $(-9, 3, 14); l \doteq 16.91$
c. $(1, 1, 0); l \doteq 1.41$
d. $(2, 0, -9); l \doteq 9.22$

5. **a.** on the xy-plane
b. on the xz-plane
c. on the yz-plane

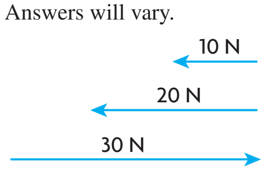
6. **a.** $\vec{i} - 7\vec{j}$
b. $6\vec{i} - 2\vec{j}$
c. $-8\vec{i} + 11\vec{j} + 3\vec{k}$
d. $4\vec{i} - 6\vec{j} + 8\vec{k}$

7. **a.** $\vec{i} + 3\vec{j} - \vec{k}$
b. $5\vec{i} + \vec{j} - \vec{k}$
c. $12\vec{i} + \vec{j} - 2\vec{k}$

Section 7.1, pp. 362–364

1. **a.** 10 N is a melon, 50 N is a chair, 100 N is a computer
b. Answers will vary.

2. **a.**



b. 180°

3. a line along the same direction

4. For three forces to be in equilibrium, they must form a triangle, which is a planar figure.

5. **a.** The resultant is 13 N at an angle of N 22.6° E. The equilibrant is 13 N at an angle of S 22.6° W.
b. The resultant is 15 N at an angle of S 36.9° W. The equilibrant is 15 N at N 36.9° E.

6. **a.** yes **b.** yes **c.** no **d.** yes

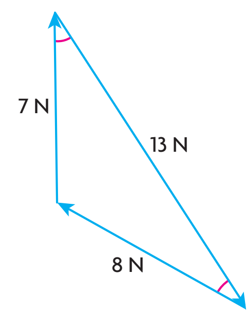
7. Arms 90 cm apart will yield a resultant with a smaller magnitude than at 30 cm apart. A resultant with a smaller magnitude means less force to counter your weight, hence a harder chin-up.

8. The resultant would be 12.17 N at 34.7° from the 6 N force toward the 8 N force. The equilibrant would be 12.17 N at 145.3° from the 6 N force away from the 8 N force.

9. 9.66 N 15° from given force, 2.95 N perpendicular to 9.66 N force

10. 49 N directed up the ramp

11. **a.**

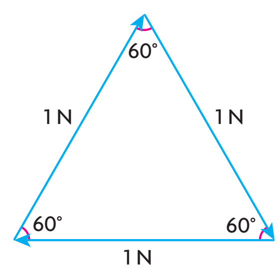


b. 60°

12. approximately 7.1 N 45° south of east

13. **a.** 7 N
b. The angle between f_1 and the resultant is 16.3°. The angle between $\vec{f_1}$ and the equilibrant is 163.7°.

14. **a.**



For these three equal forces to be in equilibrium, they must form an equilateral triangle. Since the resultant will lie along one of these lines, and since all angles of an equilateral triangle are 60°, the resultant will be at a 60° angle with the other two vectors.
b. Since the equilibrant is directed

opposite the resultant, the angle between the equilibrant and the other two vectors is $180° - 60° = 120°$.

15. 7.65 N, 67.5° from $\vec{f_2}$ toward $\vec{f_3}$

16. 45° rope: 175.73 N
30° rope: 143.48 N

17. 24 cm string: approximately 39.2 N,
32 cm string: approximately 29.4 N

18. 8.5° to the starboard side

19. **a.** magnitude for resultant and equilbrant $\doteq 13.75$ N
b. $\theta_{5N} \doteq 111.3°$, $\theta_{8N} \doteq 125.6°$, $\theta_{10N} \doteq 136.7°$

20. We know that the resultant of these two forces is equal in magnitude and angle to the diagonal line of the parallelogram formed with $\vec{f_1}$ and $\vec{f_2}$ as legs and has diagonal length $|\vec{f_1} + \vec{f_2}|$. We also know from the cosine rule that $|\vec{f_1} + \vec{f_2}|^2$
$= |\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}|\cos\phi$,
where ϕ is the supplement to θ in, our parallelogram. Since we know $\phi = 180 - \theta$, then $\cos\phi = \cos(180 - \theta) = -\cos\theta$.
Thus, we have
$$|\vec{f_1} + \vec{f_2}|^2 = |\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}|\cos\phi$$
$$|\vec{f_1} + \vec{f_2}|^2 = |\vec{f_1}|^2 + |\vec{f_2}|^2 - 2|\vec{f_1}||\vec{f_2}|\cos\phi$$
$$|\vec{f_1} + \vec{f_2}| = \sqrt{|\vec{f_1}|^2 + |\vec{f_2}|^2 + 2|\vec{f_1}||\vec{f_2}|\cos\theta}$$

Section 7.2, pp. 369–370

1. **a.** 84 km/h in the direction of the train's movement
b. 76 km/h in the direction of the train's movement

2. **a.** 500 km/h north
b. 700 km/h north

3. 304.14 km/h, W 9.5° S

4. 60° upstream

5. **a.** 2 m/s forward
b. 22 m/s in the direction of the car

6. 13 m/s, N 37.6° W

7. **a.** 732.71 km/h, N 5.5° W
b. about 732.71 km

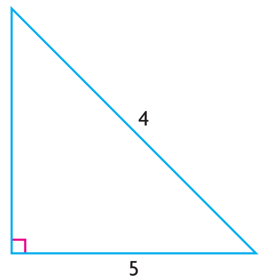
8. **a.** about 1383 km
b. about 12.5° east of north

9. **a.** about 10.4° south of west
b. 2 h

10. **a.** 5 km/h, 53.1° downstream to the bank.
b. about 0.67 km
c. 20 min

11. **a.** about 18.4° west of north
b. about 108 km/h

12.



Since her swimming speed is a maximum of 4 km/h, this is her maximum resultant magnitude, which is also the hypotenuse of the triangle formed by her and the river's velocity vector. Since one of these legs is 5 km/h, we have a triangle with a leg larger than its hypotenuse, which is impossible.

13. **a.** about 66.0 m
b. 100 s

14. **a.** about 58.5°, upstream
b. about 58.6 s

15. 35 h

Section 7.3, pp. 377–378

1. To be guaranteed that the two vectors are perpendicular, the vectors must be nonzero.

2. $\vec{a} \cdot \vec{b}$ is a scalar, and a dot product is only defined for vectors.

3. Answers may vary. For example, let $\vec{a} = \vec{i}, \vec{b} = \vec{j}, \vec{c} = -\vec{i}, \vec{a} \cdot \vec{b} = 0$, $\vec{b} \cdot \vec{c} = 0$ but $\vec{a} = -\vec{c}$.

4. $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a} = \vec{b} \cdot \vec{c}$ because $\vec{c} = \vec{a}$

5. -1

6. **a.** 16
b. -6.93
c. 0
d. -1
e. 0
f. -26.2

7. **a.** 30°
b. 80°
c. 53°
d. 127°
e. 60°
f. 120°

8. 22.5

9. **a.** $2|\vec{a}|^2 - 15|\vec{b}|^2 + 7\vec{a} \cdot \vec{b}$
b. $6|\vec{x}|^2 - 19\vec{x} \cdot \vec{y} + 3|\vec{y}|^2$

10. 0

11. 1

12. **a.** $(\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$
$= \vec{a} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} + \vec{b} \cdot \vec{b}$
$= |\vec{a}|^2 + \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} + |\vec{b}|^2$
$= |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2$
b. $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$
$= \vec{a} \cdot \vec{a} - \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{a} - \vec{b} \cdot \vec{b}$
$= |\vec{a}|^2 - \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{b} - |\vec{b}|^2$
$= |\vec{a}|^2 - |\vec{b}|^2$

13. **a.** $|\vec{a}|^2 = \vec{a} \cdot \vec{a}$
$= (\vec{b} + \vec{c}) \cdot (\vec{b} + \vec{c})$
$= |\vec{b}|^2 + 2\vec{b} \cdot \vec{c} + |\vec{c}|^2$
b. $\vec{b} \cdot \vec{c} = 0$, vectors are perpendicular. Therefore $|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2$, which is the Pythagorean theory.

14. 14

15. $|\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2$
$= (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v})$
$+ (\vec{u} - \vec{v}) \cdot (\vec{u} - \vec{v})$
$= |\vec{u}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{v}|^2 + |\vec{u}|^2$
$- 2\vec{u} \cdot \vec{v} + |\vec{v}|^2$
$= 2|\vec{u}|^2 + 2|\vec{v}|^2$

16. 3

17. -7

18. $\vec{d} = \vec{b} - \vec{c}$
$\vec{b} = \vec{d} + \vec{c}$
$\vec{c} \cdot \vec{a} = ((\vec{b} \cdot \vec{a})\vec{a}) \cdot \vec{a}$
$\vec{c} \cdot \vec{a} = (\vec{b} \cdot \vec{a})(\vec{a} \cdot \vec{a})$ because $\vec{b} \cdot \vec{a}$ is a scalar
$\vec{c} \cdot \vec{a} = (\vec{b} \cdot \vec{a})|\vec{a}|^2$
$\vec{c} \cdot \vec{a} = (\vec{d} + \vec{c}) \cdot \vec{a}$ because $|\vec{a}| = 1$
$\vec{c} \cdot \vec{a} = \vec{d} \cdot \vec{a} + \vec{c} \cdot \vec{a}$
$\vec{d} \cdot \vec{a} = 0$

Section 7.4, pp. 385–387

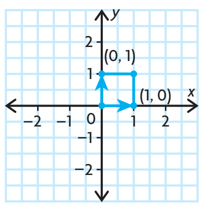
1. Any vector of the form (c, c) is perpendicular to $\vec{a}$. Therefore, there are infinitely many vectors perpendicular to $\vec{a}$. Answers may vary. For example: $(1, 1), (2, 2), (3, 3)$.

2. **a.** 0; 90°
b. $34 > 0$; acute
c. $-3 < 0$; obtuse

3. Answer may vary. For example:
a. $(0, 0, 1)$ is perpendicular to every vector in the *xy*-plane.
b. $(0, 1, 0)$ is perpendicular to every vector in the *xz*-plane.
c. $(1, 0, 0)$ is perpendicular to every vector in the *yz*-plane.

4. **a.** $(1, 2, -1)$ and $(4, 3, 10)$;
$(-4, -5, -6)$ and $\left(5, -3, -\frac{5}{6}\right)$
b. no

5. a. $\vec{a}$ and $\vec{b}$ are non-collinear and lie on the xy– plane. All vectors perpendicular to these are in R^3.
b. Any pair of non-collinear vectors in R^2 line on the xy– plane, All vectors perpendicular to these are in R^3.
6. a. about 148°
b. about 123°
c. about 64°
d. about 154°
7. a. $k = \frac{2}{3}$
b. $k \geq 0$
8. a.



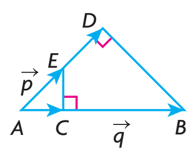
b. $(1, 1)$ and $(1, -1)$; $(1, 1)$ and $(-1, 1)$
c. $(1, 1) \cdot (1, -1)$
$= 1 - 1$
$= 0$
or
$(1, 1) \cdot (-1, 1)$
$= -1 + 1$
$= 0$
9. a. 90°
b. 30°
10. a. i. $p = \frac{8}{3}$; $q = 3$
ii. Answers may vary. For example, $p = 1$, $q = -50$.
b. Unique for collinear vectors; not unique for perpendicular vectors
11. $\theta_A = 90°$; $\theta_B \doteq 26.6°$; $\theta_C \doteq 63.4°$
12. a. $O = (0, 0, 0)$, $A = (7, 0, 0)$, $B = (7, 4, 0)$, $C = (0, 4, 0)$, $D = (7, 0, 5)$, $E = (0, 4, 5)$, $F = (0, 0, 5)$
b. 50°
13. a. Answers may vary. For example, $(3, 1, 1)$.
b. Answers may vary. For example, $(1, 1, 1)$.
14. 3 or −1
15. a. $3 + 4p + q = 0$
b. 0
16. Answers may vary. For example, $(1, 0, 1)$ and $(1, 1, 3)$.
$(x, y, z)(1, 2, -1) = 0$
$x + 2y - z = 0$
Let $x = z = 1$.
$(1, 0, 1)$ is perpendicular to $(1, 2, -1)$ and $(-2, -4, 2)$.
Let $x = y = 1$.
$(1, 1, 3)$ is perpendicular to $(1, 2, -1)$ and $(-2, -4, 2)$.
17. 4
18. a. $\vec{a} \cdot \vec{b} = 0$
Therefore, since the two diagonals are perpendicular, all the sides must be the same length.
b. $\overrightarrow{AB} = (1, 2, -1)$,
$\overrightarrow{BC} = (2, 1, 1)$,
$|\overrightarrow{AB}| = |\overrightarrow{BC}| = \sqrt{6}$
c. $\theta_1 = 60°$; $\theta_2 = 120°$
19. a. $(6, 18, -4)$
b. 87.4°
20. $\alpha \doteq 109.5°$ or $\theta \doteq 70.5°$

Mid-Chapter Review, pp. 388–389

1. a. 3
b. 81
2. 15 cm cord: 117.60 N;
20 cm cord: 88.20 N
3. 0
4. a. about 575.1 km/h at S 7.06° E
b. about 1.74 h
5. a. about 112.61 N
b. about 94.49 N
6. 4.5
7. a. 34
b. $\frac{34}{63}$
8. a. 0
b. 5
c. $5\vec{i} - 4\vec{j} + 3\vec{k}$
d. 0
e. 34
f. 9
9. a. $x = -3$ or $x = -\frac{1}{3}$
b. no value
10. a. $\vec{i} - 4\vec{j} - \vec{k}$
b. 24
c. $\sqrt{2}$ or 1.41
d. −4
e. −12
11. about 126.9°
12. $\vec{F} \doteq 6.08$ N, 25.3° from the 4 N force towards the 3 N force. $\vec{E} \doteq 6.08$ N, $180° - 25.3° = 154.7°$ from the 4 N force away from the 3 N force.
13. a. about 109.1°
b. about 87.9°
14. a. about N 2.6° E
b. about 2.17 h
15. $\vec{x} = \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ or $\left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right)$
16. a. about 61.2 m
b. about 84.9 s
17. a. when $\vec{x}$ and $\vec{y}$ have the same length
b. Vectors $\vec{a}$ and $\vec{b}$ determine a parallelogram. Their sum $\vec{a} + \vec{b}$ is one diagonal of the parallelogram formed, with its tail in the same location as the tails of $\vec{a}$ and $\vec{b}$. Their difference $\vec{a} - \vec{b}$ is the other diagonal of the parallelogram.
18. about 268.12 N

Section 7.5, pp. 398–400

1. a. scalar projection $= 2$,
vector projection $= 2\vec{i}$
b. scalar projection $= 3$,
vector projection $= 3\vec{j}$
2. Using the formula would cause a division by 0. Generally the $\vec{0}$ has any direction and 0 magnitude. You cannot project onto nothing.
3. You are projecting $\vec{a}$ onto the tail of $\vec{b}$, which is a point with magnitude 0. Therefore, it is $\vec{0}$; the projections of $\vec{b}$ onto the tail of $\vec{a}$ are also 0 and $\vec{0}$.
4. Answers may vary. For example, $\vec{p} = \overrightarrow{AE}$, $\vec{q} = \overrightarrow{AB}$

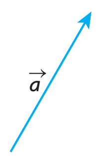


scalar projection $\vec{p}$ on $\vec{q} = |\overrightarrow{AC}|$,
vector projection $\vec{p}$ on $\vec{q} = \overrightarrow{AC}$,
scalar projection $\vec{q}$ on $\vec{p} = |\overrightarrow{AD}|$,
vector projection $\vec{q}$ on $\vec{p} = \overrightarrow{AD}$
5. scalar projection of $\vec{a}$ on $\vec{i} = -1$,
vector projection of $\vec{a}$ on $\vec{i} = -\vec{i}$,
scalar projection of $\vec{a}$ on $\vec{j} = 2$,
vector projection of $\vec{a}$ on $\vec{j} = 2\vec{j}$,
scalar projection of $\vec{a}$ on $\vec{k} = -5$,
vector projection of $\vec{a}$ on $\vec{k} = -5\vec{k}$;
Without having to use formulae, a projection of $(-1, 2, 5)$ on $\vec{i}$, $\vec{j}$, or $\vec{k}$ is the same as a projection of $(-1, 0, 0)$ on $\vec{i}$, $(0, 2, 0)$ on $\vec{j}$, and $(0, 0, 5)$ on $\vec{k}$, which intuitively yields the same result.
6. a. scalar projection: $\frac{\vec{p} \cdot \vec{q}}{|\vec{q}|} = \frac{458}{21}$,
vector projection: $\frac{458}{441}(-4, 5, -20)$
b. about 82.5°, about 74.9°, about 163.0°

7. **a.** scalar projection: 0,
vector projection: $\vec{0}$
b. scalar projection: 2,
vector projection: $2\vec{i}$
c. scalar projection: $\frac{50}{13}$,
vector projection: $\frac{50}{169}(-5, 12)$

8. **a.** The scalar projection of $\vec{a}$ on the x-axis $(X, 0, 0)$ is -1; The vector projection of $\vec{a}$ on the x-axis is $-\vec{i}$; The scalar projection of $\vec{a}$ on the y-axis $(0, Y, 0)$ is 2; The vector projection of $\vec{a}$ on the y-axis is $2\vec{j}$; The scalar projection of $\vec{a}$ on the z-axis $(0, 0, Z)$ is 4; The vector projection of $\vec{a}$ on the z-axis is $4\vec{k}$.
b. The scalar projection of $m\,\vec{a}$ on the x-axis $(X, 0, 0)$ is $-m$; The vector projection of $m\,\vec{a}$ on the x-axis is $-m\vec{i}$; The scalar projection of $m\,\vec{a}$ on the y-axis $(0, Y, 0)$ is $2m$; The vector projection $m\,\vec{a}$ on the y-axis $(0, Y, 0)$ is $2m\vec{j}$; The scalar projection of $m\,\vec{a}$ on the z-axis $(0, 0, Z)$ is $4m$; The vector projection of $m\,\vec{a}$ on the z-axis is $4m\vec{k}$.

9. **a.**



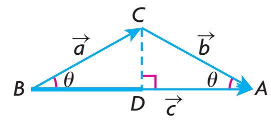
vector projection: $\vec{a}$
scalar projection: $|\vec{a}|$
b. $|\vec{a}|\cos\theta = |\vec{a}|\cos 0$
$= |\vec{a}|(1)$
$= |\vec{a}|$.
The vector projection is the scalar projection multiplied by $\frac{\vec{a}}{|\vec{a}|}$,
$|\vec{a}| \times \frac{\vec{a}}{|\vec{a}|} = \vec{a}$.

10. **a.**

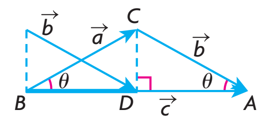
b. $\frac{(-\vec{a})\cdot\vec{a}}{|\vec{a}|} = \frac{-|\vec{a}|^2}{|\vec{a}|}$
$= -|\vec{a}|$
So, the vector projection is
$-|\vec{a}|\left(\frac{\vec{a}}{|\vec{a}|}\right) = -\vec{a}$.

11. **a.** scalar projection of $\overrightarrow{AB}$ on the x-axis is -2; vector projection of $\overrightarrow{AB}$ on the x-axis is $-2\vec{i}$; scalar projection of $\overrightarrow{AB}$ on the y-axis is 1; vector projection of $\overrightarrow{AB}$ on the y-axis is $\vec{j}$; scalar projection of $\overrightarrow{AB}$ on the z-axis is 2; vector projection of $\overrightarrow{AB}$ on the z-axis is $2\vec{k}$.
b. 70.5°

12. **a.** $|\overrightarrow{BD}|$

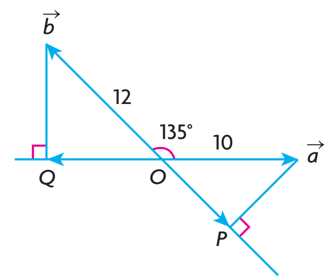


b. $|\overrightarrow{BD}|$



c. In an isosceles triangle, CD is a median and a right bisector of BA.
d. Yes

13. **a.** -7.07, -8.49
b.



$\overrightarrow{OQ}$ is the vector projection of $\vec{b}$ on $\vec{a}$
$\overrightarrow{OP}$ is the vector projection of $\vec{a}$ on $\vec{b}$

14. **a.** $-\frac{1}{3}$
b. The scalar projection of $\overrightarrow{BC}$ on $\overrightarrow{OD}$ is $\frac{19}{3}\cdot -\frac{1}{3} + \frac{19}{13} = 6$
The scalar projection of $\overrightarrow{AC}$ on $\overrightarrow{OD}$ is 6.
c. Same lengths and both are in the direction of $\overrightarrow{OD}$. Add to get one vector.

15. **a.** $1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$

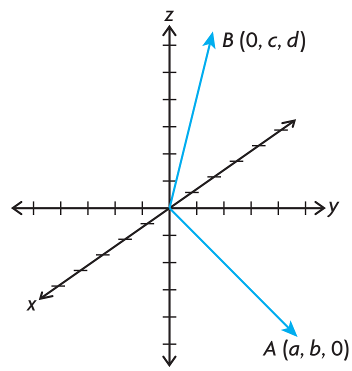
$$= \left(\frac{a}{\sqrt{a^2+b^2+c^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2+c^2}}\right)^2 + \left(\frac{c}{\sqrt{a^2+b^2+c^2}}\right)^2$$
$$= \frac{a^2}{a^2+b^2+c^2} + \frac{b^2}{a^2+b^2+c^2} + \frac{c^2}{a^2+b^2+c^2}$$
$$= \frac{a^2+b^2+c^2}{a^2+b^2+c^2}$$
$$= 1$$

b. Answers may vary. For example: $(0, \frac{\sqrt{3}}{2}, \frac{1}{2})$, $(0, \sqrt{3}, 1)$
c. If two angles add to 90°, then all three will add to 180°.

16. **a.** about 54.7°
b. about 125.3°

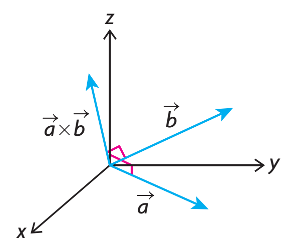
17. $\cos^2 x + \sin^2 x = 1$
$\cos^2 x = 1 - \sin^2 x$
$1 = \cos^2\alpha + \cos^2\beta + \cos^2\gamma$
$1 = (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma)$
$1 = 3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma)$
$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

18. Answers may vary. For example:

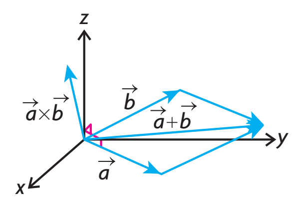


Section 7.6, pp. 407–408

1. **a.**



$\vec{a} \times \vec{b}$ is perpendicular to $\vec{a}$. Thus, their dot product must equal 0. The same applies to the second case.



b. $\vec{a} + \vec{b}$ is still in the same plane formed by $\vec{a}$ and $\vec{b}$, thus $\vec{a} + \vec{b}$ is perpendicular to $\vec{a} \times \vec{b}$ making the dot product 0 again.
c. Once again, $\vec{a} - \vec{b}$ is still in the same plane formed by $\vec{a}$ and $\vec{b}$, thus $\vec{a} - \vec{b}$ is perpendicular to $\vec{a} \times \vec{b}$ making the dot product 0 again.

2. $\vec{a} \times \vec{b}$ produces a vector, not a scalar. Thus, the equality is meaningless.

3. a. It's possible because there is a vector crossed with a vector, then dotted with another vector, producing a scalar.
b. This is meaningless because $\vec{a} \cdot \vec{b}$ produces a scalar. This results in a scalar crossed with a vector, which is meaningless.
c. This is possible. $\vec{a} \times \vec{b}$ produces a vector, and $\vec{c} + \vec{d}$ also produces a vector. The result is a vector dotted with a vector producing a scalar.
d. This is possible. $\vec{a} \cdot \vec{b}$ produces a scalar, and $\vec{c} \times \vec{d}$ produces a vector. The product of a scalar and vector produces a vector.
e. This is possible. $\vec{a} \times \vec{b}$ produces a vector, and $\vec{c} \times \vec{d}$ produces a vector. The cross product of a vector and vector produces a vector.
f. This is possible. $\vec{a} \times \vec{b}$ produces a vector. When added to another vector, it produces another vector.

4. a. $(-7, -8, -2)$
b. $(1, 5, 1)$
c. $(-11, -33, 22)$
d. $(-19, -22, 7)$
e. $(3, 3, -1)$
f. $(-8, -26, 11)$

5. 1

6. a. $(-4, 0, 0)$
b. Vectors of the form $(0, b, c)$ are in the yz-plane. Thus, the only vectors perpendicular to the yz-plane are those of the form $(a, 0, 0)$ because they are parallel to the x-axis.

7. a. $(1, 2, 1) \times (2, 4, 2)$
$= (2(2) - 1(4), 1(2) - 1(2), 1(4) - 2(2))$
$= (0, 0, 0)$
b. $(a, b, c) \times (ka, kb, kc)$
$= (b(kc) - c(kb), c(ka) - a(kc), a(kb) - b(ka))$
Using the associative law of multiplication, we can rearrange this:
$= (bck - bck, ack - ack, abk - abk)$
$= (0, 0, 0)$

8. a. $\vec{p} \times (\vec{q} + \vec{r}) = (-26, -7, 3)$
$\vec{p} \times \vec{q} + \vec{p} \times \vec{r} = (-26, -7, 3)$
b. $\vec{p} \times (\vec{q} + \vec{r}) = (-3, 2, 5)$
$\vec{p} \times \vec{q} + \vec{p} \times \vec{r} = (-3, 2, 5)$

9. a. $\vec{i} \times \vec{j} = (1, 0, 0) \times (0, 1, 0) = \vec{k}$
$-\vec{j} \times \vec{i} = (0, -1, 0) \times (1, 0, 0) = \vec{k}$
b. $\vec{j} \times \vec{k} = (0, 1, 0) \times (0, 0, 1) = \vec{i}$
$-\vec{k} \times \vec{j} = (0, 0, -1) \times (0, 1, 0) = \vec{i}$
c. $\vec{k} \times \vec{i} = (0, 0, 1) \times (1, 0, 0) = \vec{j}$
$-\vec{i} \times \vec{k} = (-1, 0, 0) \times (0, 0, 1) = \vec{j}$

10. $k(a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)(a_1, a_2, a_3)$
$= k(a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_2a_1b_3 + a_3a_1b_2 - a_3a_2b_1)$
$= k(0)$
$= 0$
$\vec{a}$ is perpendicular to $k(\vec{a} \times \vec{b})$.

11. a. $(0, 0, 6), (0, 0, -6)$
b. $(0, 0, 0)$
c. All the vectors are in the xy-plane. Thus, their cross product in part b. is between vectors parallel to the z-axis and so parallel to each other. The cross product of parallel vectors is $\vec{0}$.

12. Let $\vec{x} = (1, 0, 1)$, $\vec{y} = (1, 1, 1)$, and $\vec{z} = (1, 2, 3)$
Then, $\vec{x} \times \vec{y} = (0 - 1, 1 - 1, 1 - 0)$
$= (-1, 0, 1)$
$(\vec{x} \times \vec{y}) \times \vec{z}$
$= (0 - 2, 1 - (-3), -3 - 0)$
$= (-2, 4, -3)$
$\vec{y} \times \vec{z} = (3 - 2, 1 - 3, 2 - 1)$
$= (1, -2, 1)$
$\vec{x} \times (\vec{y} \times \vec{z}) = (0 + 2, 1 - 1, -2 - 0)$
$= (2, 0, -2)$
Thus, $(\vec{x} \times \vec{y}) \times \vec{z} \neq \vec{x} \times (\vec{y} \times \vec{z})$.

13. By the distributive property of cross product:
$= (\vec{a} - \vec{b}) \times \vec{a} + (\vec{a} - \vec{b}) \times \vec{b}$
By the distributive property again:
$= \vec{a} \times \vec{a} - \vec{b} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{b}$
A vector crossed with itself equals $\vec{0}$, thus:
$= -\vec{b} \times \vec{a} + \vec{a} \times \vec{b}$
$= \vec{a} \times \vec{b} - \vec{b} \times \vec{a}$
$= \vec{a} \times \vec{b} - (-\vec{a} \times \vec{b})$
$= 2\vec{a} \times \vec{b}$

Section 7.7, pp. 414–415

1. By pushing as far away from the hinge as possible, $|\vec{r}|$ is increased, making the cross product bigger. By pushing at right angles, sine is its largest value, 1, making the cross product larger.

2. a. 0
b. This makes sense because the vectors lie on the same line. Thus, the parallelogram would just be a line making its area 0.

3. a. 450 J
b. about 10 078.91 J
c. about 32 889.24 J
d. 35 355.34 J

4. a. $\vec{k}$
b. $-\vec{k}$
c. $-\vec{j}$
d. $\vec{j}$

5. a. $\sqrt{3}$ square units
b. $\sqrt{213}$ square units

6. $2, \frac{-12}{5}$

7. a. $\frac{5\sqrt{6}}{2}$ square units
b. $\frac{5\sqrt{6}}{2}$ square units.
c. Any two sides of a triangle can be used to calculate its area.

8. about 0.99 J

9. $\frac{6}{\sqrt{26}}$ or about 1.18

10. a. $\vec{p} \times \vec{q} = (-6 - 3, 6 - 3, 1 + 4)$
$= (-9, 3, 5)$
$(\vec{p} \times \vec{q}) \times \vec{r}$
$= (0 - 5, 5 + 0, -9 - 3)$
$= (-5, 5, -12)$
$a(1, -2, 3) + b(2, 1, 3)$
$= (-5, 5, -12)$
Looking at x-components:
$a + 2b = -5$
$a = -5 - 2b$
y-components:
$-2a + b = 5$
Substitute a:
$10 + 4b + b = 5$
$5b = -5$
$b = -1$
Substitute b back into the x-components:
$a = -5 + 2$
$a = -3$
Check in z-components:
$3a + 3b = -12$
$-9 - 3 = -12$
b. $\vec{p} \cdot \vec{r} = 1 - 2 + 0 = -1$
$\vec{q} \cdot \vec{r} = 2 + 1 + 0 = 3$
$(\vec{p} \cdot \vec{r})\vec{q} - (\vec{q} \cdot \vec{r})\vec{p}$
$= -1(2, 1, 3) - 3(1, -2, 3)$
$= (-2, -1, -3) - (3, -6, 9)$
$= (-2 - 3, -1 + 6, -3 - 9)$
$= (-5, 5, -12)$

Review Exercise, pp. 418–421

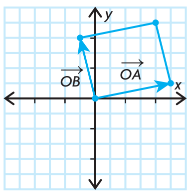
1. a. $(2, 0, 2)$
b. $(-4, 0, -4)$
c. 16
d. The cross products are parallel, so the original vectors are in the same plane.

2. **a.** 3
b. 7
c. $4\sqrt{3}$
d. $2\sqrt{17}$
e. 5
f. -1

3. **a.** 6
b. $-\frac{54}{5}$

4. about 18.52°

5. **a.**

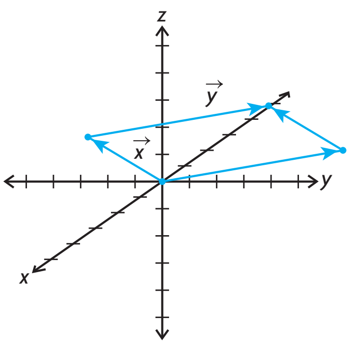


b. about 77.9°

6. rope at 45°: about 87.86 N, rope at 30°: about 71.74 N

7. 304.14 km/h, W 9.46° N

8. **a.**



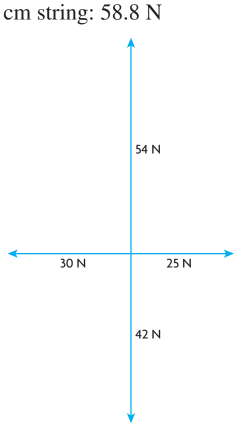
b. approximately 56.78

9. $\left(\frac{9}{\sqrt{115}}, -\frac{5}{\sqrt{115}}, -\frac{3}{\sqrt{115}}\right)$

10. **a.** about 77.64° is the largest angle
b. 36.50

11. 30 cm string: 78.4 N; 40 cm string: 58.8 N

12. **a.**



b. The resultant is 13 N in a direction N22.6°W. The equilibrant is 13 N in a direction S22.6°E.

13. **a.** Let D be the origin, then:
$A = (2, 0, 0), B = (2, 4, 0),$
$C = (0, 4, 0), D = (0, 0, 0),$
$E = (2, 0, 3), F = (2, 4, 3),$
$G = (0, 4, 3), H = (0, 0, 3)$
b. about 44.31°
c. about 3.58

14. 7.5

15. **a.** about 48.2°
b. about 8 min 3 s
c. Such a situation would have resulted in a right triangle where one of the legs is longer than the hypotenuse, which is impossible.

16. **a.** $\overrightarrow{OA} + \overrightarrow{OB} = (-3, 8, -8),$
$\overrightarrow{OA} - \overrightarrow{OB} = (9, -4, -4)$
b. about 84.36°

17. **a.** $a = -4$ and $b = 4$
b. $a = b + 9$
c. $\left(\frac{1}{\sqrt{5}}, 0, \frac{2}{3\sqrt{5}}\right)$

18. **a.** about 74.62°
b. about 0.75
c. $(0.1875)(\sqrt{3}, -2, -3)$
d. about 138.59°

19. **a.** special
b. not special

20. **a.** $(-1, 1, 3)$
b. $(-2, 2, 6)$
c. 0
d. $(-5, -2, -1)$

21. about 11.55 N

22. $(2, -8, -10)$

23. -141

24. 5 or -7

25. about 103.34°

26. **a.** $C = (3, 0, 5), F = (0, 4, 0)$
b. $(-3, 4, -5)$
c. about 111.1°

27. **a.** about 7.30
b. about 3.84
c. about 3.84

28. **a.** scalar: 1, vector: $\vec{i}$
b. scalar: 1, vector: $\vec{j}$
c. scalar: $\frac{1}{\sqrt{2}}$, vector: $\frac{1}{2}(\vec{k} + \vec{j})$

29. **a.** $\vec{b}, \vec{c}$
b. $\vec{a}$; When dotted with $\vec{d}$, it equals 0.

30. 7.50 J

31. **a.** $\vec{a} \cdot \vec{b} = 6 - 5 - 1 = 0$
b. $\vec{a}$ with the x-axis:
$|\vec{a}| = \sqrt{4 + 25 + 1} = \sqrt{30}$
$\cos(\alpha) = \frac{2}{\sqrt{30}}$
$\vec{a}$ with the y-axis:
$\cos(\beta) = \frac{5}{\sqrt{30}}$
$\vec{a}$ with the z-axis:
$\cos(\gamma) = \frac{-1}{\sqrt{30}}$
$|\vec{b}| = \sqrt{9 + 1 + 1} = \sqrt{11}$
$\vec{b}$ with the x-axis:
$\cos(\alpha) = \frac{3}{\sqrt{11}}$
$\vec{b}$ with the y-axis:
$\cos(\beta) = \frac{-1}{\sqrt{11}}$
$\vec{b}$ with the z-axis:
$\cos(\gamma) = \frac{1}{\sqrt{11}}$
c. $\vec{m}_1 \times \vec{m}_2 = \frac{6}{\sqrt{330}} - \frac{5}{\sqrt{330}} - \frac{1}{\sqrt{330}} = 0$

32. $|3\vec{i} + 3\vec{j} + 10\vec{k}| = \sqrt{118}$
$|-\vec{i} + 9\vec{j} - 6\vec{k}| = \sqrt{118}$

33. **a.** $\cos\alpha = \frac{\sqrt{3}}{2},$
$\cos\beta = \cos\gamma = \pm\frac{1}{2\sqrt{2}}$
b. acute case: 69.3°, obtuse case: 110.7°

34. -5

35. $\vec{a} \cdot \vec{b} = 0 - 20 + 12 = -8$
$|\vec{a} + \vec{b}| = \sqrt{1 + 1 + 64} = \sqrt{66}$
$|\vec{a} - \vec{b}| = \sqrt{1 + 81 + 16} = \sqrt{98}$
$\frac{1}{4}|\vec{a} + \vec{b}|^2 - \frac{1}{4}|\vec{a} - \vec{b}|^2$
$= \frac{66}{4} - \frac{98}{4} = -8$

36. $\vec{c} = \vec{b} - \vec{a}$
$|\vec{c}|^2 = |\vec{b} - \vec{a}|^2$
$= (\vec{b} - \vec{a})(\vec{b} - \vec{a})$
$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a} \cdot \vec{b}$
$= |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos\theta$

37. $\overrightarrow{AB} = (2, 0, 4)$
$|\overrightarrow{AB}| = 2\sqrt{5}$
$\overrightarrow{AC} = (1, 0, 2)$
$|\overrightarrow{AC}| = \sqrt{5}$
$\overrightarrow{BC} = (-1, 0, -2)$
$|\overrightarrow{BC}| = \sqrt{5}$

$\cos A = 1$
$\cos B = 1$
$\cos C = -1$
area of triangle $ABC = 0$

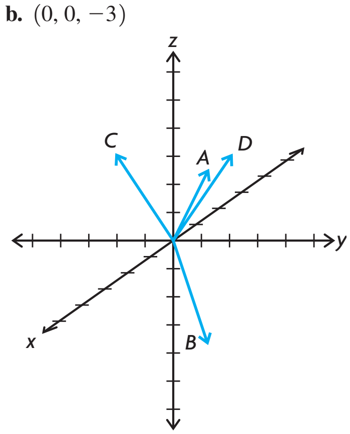
Chapter 7 Test, p. 422

1. **a.** $(-4, -1, -3)$
 b. $(-4, -1, -3)$
 c. 0
 d. $(0, 0, 0)$
2. **a.** scalar projection: $\frac{1}{3}$,
 vector projection: $\frac{1}{9}(2, -1, -2)$.
 b. x-axis: 48.2°; y-axis: 109.5°; z-axis: 131.8°
 c. $\sqrt{26}$ or 5.10
3. Both forces have a magnitude of 78.10 N. The resultant makes an angle 33.7° to the 40 N force and 26.3° to the 50 N force. The equilibrant makes an angle 146.3° to the 40 N force and 153.7° to the 50 N force.
4. 1004.99 km/h, N 5.7° W
5. **a.** 96 m downstream
 b. 28.7° upstream
6. 3.50 square units.
7. cord at 45°: about 254.0 N;
 cord at 70°: about 191.1 N
8. **a.** $\vec{x} \cdot \vec{y}$
 $|\vec{x} + \vec{y}| = 33$
 $|\vec{x} - \vec{y}| = 33$
 $\frac{1}{4}|\vec{x} + \vec{y}|^2 - \frac{1}{4}|\vec{x} - \vec{y}|^2$
 $= \frac{1}{4}(33) - \frac{1}{4}(33) = 0$
 So, the equation holds for these vectors.
 b. $|\vec{x} + \vec{y}|^2 = (\vec{x} + \vec{y})(\vec{x} + \vec{y})$
 $= (\vec{x} \cdot \vec{x}) + (\vec{x} \cdot \vec{y}) + (\vec{y} \cdot \vec{x}) + (\vec{y} \cdot \vec{y})$
 $= (\vec{x} \cdot \vec{x}) + 2(\vec{x} \cdot \vec{y}) + (\vec{y} \cdot \vec{y})$
 $|\vec{x} - \vec{y}|^2 = (\vec{x} - \vec{y})(\vec{x} - \vec{y})$
 $= (\vec{x} \cdot \vec{x}) + (\vec{x} \cdot -\vec{y}) + (-\vec{y} \cdot \vec{x}) + (-\vec{y} \cdot -\vec{y})$
 $= (\vec{x} \cdot \vec{x}) - 2(\vec{x} \cdot \vec{y}) + (\vec{y} \cdot \vec{y})$
 So, the right side of the equation is
 $\frac{1}{4}|\vec{x} + \vec{y}|^2 - \frac{1}{4}|\vec{x} - \vec{y}|^2$
 $= \frac{1}{4}(4(\vec{x} \cdot \vec{y}))$
 $= \vec{x} \cdot \vec{y}$

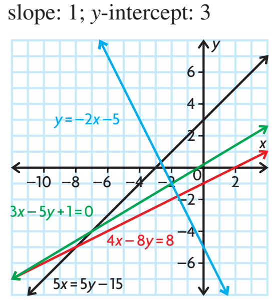
Chapter 8

Review of Prerequisite Skills, pp. 424–425

1. **a.** $(2, -9, 6)$
 b. $(13, -12, -41)$
2. **a.** yes **c.** yes
 b. yes **d.** no
3. yes
4. $t = 18$
5. **a.** $(3, 1)$
 b. $(5, 6)$
 c. $(-4, 7, 0)$
6. 52.4
7. **a.** $(-22, -8, -13)$
 b. $(0, 0, -3)$
8.



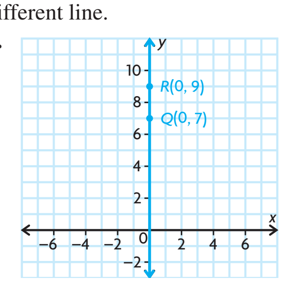
9. **a.** $(-7, -3)$
 b. $(10, 14)$
 c. $(2, -8, 5)$
 d. $(-4, 5, 4)$
10. **a.** $(7, 3)$
 b. $(-10, -14)$
 c. $(-2, 8, -5)$
 d. $(4, -5, -4)$
11. **a.** slope: -2; y-intercept: -5
 b. slope: $\frac{1}{2}$; y-intercept: -1
 c. slope: $\frac{3}{5}$; y-intercept: $\frac{1}{5}$
 d. slope: 1; y-intercept: 3



12. Answers may vary. For example:
 a. $(8, 14)$
 b. $(-15, 12, 9)$
 c. $\vec{i} + 3\vec{j} - 2\vec{k}$
 d. $-20\vec{i} + 32\vec{j} + 8\vec{k}$
13. **a.** 33
 b. -33
 c. 77
 d. $(-11, -8, 28)$
 e. $(11, 8, -28)$
 f. $(55, 40, -140)$
14. The dot product of two vectors yields a real number, while the cross product of two vectors gives another vector.

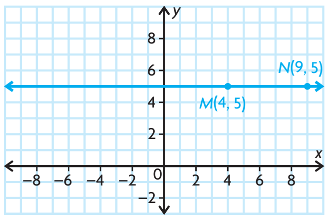
Section 8.1, pp. 433–434

1. Direction vectors for a line are unique only up to scalar multiplication. So, since each of the given vectors is just a scalar multiple of $\left(\frac{1}{3}, \frac{1}{6}\right)$, each is an acceptable direction vector for the line.
2. **a.** Answers may vary. For example, $(-2, 7)$, $(1, 5)$, and $(4, 3)$.
 b. $t = -5$
 If $t = -5$, then $x = -14$ and $y = 15$. So $P(-14, 15)$ is a point on the line.
3. Answers may vary. For example:
 a. direction vector: $(2, 1)$; point: $(3, 4)$
 b. direction vector: $(2, -7)$; point: $(1, 3)$
 c. direction vector: $(0, 2)$; point: $(4, 1)$
 d. direction vector: $(-5, 0)$; point: $(0, 6)$
4. Answers may vary. For example:
 $\vec{r} = (2, 1) + t(-5, 4), t \in \mathbf{R}$
 $\vec{q} = (-3, 5) + s(5, -4), s \in \mathbf{R}$
5. **a.** $R(-9, 18)$ is a point on the line. When $t = 7$, $x = -9$ and $y = 18$.
 b. Answers may vary. For example:
 $\vec{r} = (-9, 18) + t(-1, 2), t \in \mathbf{R}$
 c. Answers may vary. For example:
 $\vec{r} = (-2, 4) + t(-1, 2), t \in \mathbf{R}$
6. Answers may vary. For example:
 a. $(-3, -4)$, $(0, 0)$, and $(3, 4)$
 b. $\vec{r} = t(1, 1), t \in \mathbf{R}$
 c. This describes the same line as part a.
7. One can multiply a direction vector by a constant to keep the same line, but multiplying the point yields a different line.
8. **a.**



 b. $\vec{r} = (0, 7) + t(0, 2), t \in \mathbf{R}$; $x = 0$, $y = 7 + 2t, t \in \mathbf{R}$

9. a.



b. $\vec{r} = (4, 5) + t(5, 0), t \in \mathbf{R}$;
$x = 4 + 5t, y = 5, t \in \mathbf{R}$

10. a. $\vec{r} = (2, 0) + t(5, -3), t \in \mathbf{R}$
b. $(0, 1.2)$

11. 9

12. First, all the relevant vectors are found.
$\overrightarrow{AB} = (-2, 3)$
$\overrightarrow{AC} = (-4, 6)$
$\overrightarrow{AD} = (-6, 9)$
a. $\overrightarrow{AC} = (-4, 6) = 2(-2, 3) = 2\overrightarrow{AB}$
b. $\overrightarrow{AD} = (-6, 9) = 3(-2, 3) = 3\overrightarrow{AB}$
c. $\overrightarrow{AC} = (-4, 6) = \frac{2}{3}(-6, 9) = \frac{2}{3}\overrightarrow{AD}$

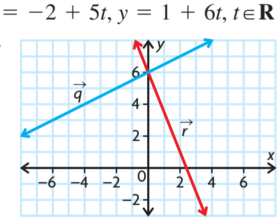
13. a. $A(5, 12)$; $B(-12, -5)$
b. $\sqrt{578}$ or about 24.04

14. In the parametric form, the second equation becomes $x = 1 + 6t$, $y = 6 + 4t, t \in \mathbf{R}$. If t is solved for in this equation, we obtain $t = \frac{x - 1}{6}$ and $t = \frac{y - 6}{4}$. Setting these two expressions equal to each other, the line is described by $\frac{x - 1}{6} = \frac{y - 6}{4}$, or by simplifying, $y - 6 = \frac{2}{3}x - \frac{2}{3}$. So, the second equation describes a line with a slope of $\frac{2}{3}$. If y is solved for in the first expression, we see that $y = \frac{2}{3}x + 5$. $(1, 6)$ is on the second line but not the first. Hence, both equations are lines with slope of $\frac{2}{3}$ and must be parallel.

Section 8.2, pp. 443–444

1. a. $\vec{m} = (6, -5)$
b. $\vec{n} = (5, 6)$
c. $(0, 9)$
d. $\vec{r} = (7, 9) + t(6, -5), t \in \mathbf{R}$;
$x = 7 + 6t, y = 9 - 5t, t \in \mathbf{R}$
e. $\vec{r} = (-2, 1) + t(5, 6), t \in \mathbf{R}$;
$x = -2 + 5t, y = 1 + 6t, t \in \mathbf{R}$

2. a.–b.



c. It produces a different line.

3. a. $\vec{r} = (0, -6) + t(8, 7), t \in \mathbf{R}$;
$x = 8t, y = -6 + 7t, t \in \mathbf{R}$
b. $\vec{r} = (0, 5) + t(2, 3), t \in \mathbf{R}$;
$x = 2t, y = 5 + 3t, t \in \mathbf{R}$
c. $\vec{r} = (0, -1) + t(1, 0), t \in \mathbf{R}$;
$x = t, y = -1, t \in \mathbf{R}$
d. $\vec{r} = (4, 0) + t(0, 1), t \in \mathbf{R}$;
$x = 4, y = t, t \in \mathbf{R}$

4. If the two lines have direction vectors that are collinear and share a point in common, then the two lines are coincident. In this example, both have $(3, 2)$ as a parallel direction vector and both have $(-4, 0)$ as a point on the line. Hence, the two lines are coincident.

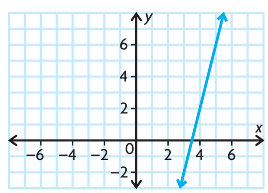
5. a. The normal vectors for the lines are $(2, -3)$ and $(4, -6)$, which are collinear. Since in two dimensions, any two direction vectors perpendicular to $(2, -3)$ are collinear, the lines have collinear direction vectors. Hence, the lines are parallel.
b. $k = 12$

6. $4x + 5y - 21 = 0$

7. $x + y - 2 = 0$

8. $2x + y - 16 = 0$

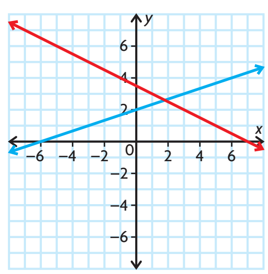
9. a.



b. $4x - y - 14 = 0$

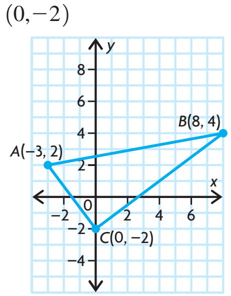
10. a. 82° **c.** 63° **e.** 54°
b. 42° **d.** 37° **f.** 63°

11. a.



b. acute: 45°, obtuse: 135°

12. a. $(0, -2)$
b.



c. $\overrightarrow{CA} = (-3 - 0, 2 - (-2))$
$= (-3, 4)$
$\overrightarrow{CB} = (8 - 0, 4 - (-2))$
$= (8, 6)$
$\overrightarrow{CA} \cdot \overrightarrow{CB} = (-3)(8) + (4)(6)$
$= -24 + 24$
$= 0$
Since the dot product of the vectors is 0, the vectors are perpendicular, and $\angle ACB = 90°$.

13. The sum of the interior angles of a quadrilateral is 360°. The normals make 90° angles with their respective lines at A and C. The angle of the quadrilateral at B is $180° - \theta$. Let x represent the measure of the interior angle of the quadrilateral at O.
$$90° + 90° + 180° - \theta + x = 360°$$
$$360° - \theta + x = 360°$$
$$x = \theta$$
Therefore, the angle between the normals is the same as the angle between the lines.

14. $2 \pm \sqrt{3}$

Section 8.3, pp. 449–450

1. a. $(-3, 1, 8)$
b. $(1, -1, 3)$
c. $(-2, 1, 3)$
d. $(-2, -3, 1)$
e. $(3, -2, -1)$
f. $\left(\frac{1}{3}, -\frac{3}{4}, \frac{2}{5}\right)$

2. a. $(-1, 1, 9)$
b. $(2, 1, -1)$
c. $(3, -4, -1)$
d. $(-1, 0, 2)$
e. $(0, 0, 2)$
f. $(2, -1, 2)$

3. a. $\vec{r} = (-1, 2, 4) + t(4, -5, 1), t \in \mathbf{R}$;
$\vec{q} = (3, -3, 5) + s(-4, 5, -1), s \in \mathbf{R}$
b. $x = -1 + 4t, y = 2 - 5t$,
$z = 4 + t, t \in \mathbf{R}$; $x = 3 - 4s$,
$y = -3 + 5s, z = 5 - s, s \in \mathbf{R}$

4. a. $\vec{r} = (-1, 5, -4) + t(1, 0, 0), t \in \mathbf{R}$
b. $x = -1 + t, y = 5, z = -4, t \in \mathbf{R}$
c. Since two of the coordinates in the direction vector are zero, a symmetric equation cannot exist.

5. a. $\vec{r} = (-1, 2, 1) + t(3, -2, 1), t \in \mathbf{R}$;
$x = -1 + 3t, y = 2 - 2t$,
$z = 1 + t, t \in \mathbf{R}$;
$\frac{x + 1}{3} = \frac{y - 2}{-2} = \frac{z - 1}{1}$

b. $\vec{r} = (-1, 1, 0) + t(0, 1, 1), t \in \mathbf{R}$;
$x = -1, y = 1 + t, z = t, t \in \mathbf{R}$;
$\frac{y-1}{1} = \frac{z}{1}, x = -1$

c. $\vec{r} = (-2, 3, 0) + t(0, 1, 1), t \in \mathbf{R}$;
$x = -2, y = 3 + t, z = t, t \in \mathbf{R}$;
$\frac{y-3}{1} = \frac{z}{1}, x = -2$

d. $\vec{r} = (-1, 0, 0) + t(0, 1, 0), t \in \mathbf{R}$;
$x = -1, y = t, z = 0, t \in \mathbf{R}$;
Since two of the coordinates in the direction vector are zero, there is no symmetric equation for this line.

e. $\vec{r} = t(-4, 3, 0), t \in \mathbf{R}$;
$x = -4t, y = 3t, z = 0, t \in \mathbf{R}$;
$\frac{x}{-4} = \frac{y}{3}, z = 0$

f. $\vec{r} = (1, 2, 4) + t(0, 0, 1), t \in \mathbf{R}$;
$x = 1, y = 2, z = 4 + t, t \in \mathbf{R}$;
Since two of the coordinates in the direction vector are zero, there is no symmetric equation for this line.

6. **a.** $x = -6 + t, y = 10 - t,$
$z = 7 + t, t \in \mathbf{R}$;
$x = -7 + s, y = 11 - s,$
$z = 5, s \in \mathbf{R}$

b. about 35.3°

7. The directional vector of the first line is $(8, 2, -2) = -2(-4, -1, 1)$. So, $(-4, -1, 1)$ is a directional vector for the first line as well. Since $(-4, -1, 1)$ is also the directional vector of the second line, the lines are the same if the lines share a point. (1, 1, 3) is a point on the second line. Since
$1 = \frac{1+7}{8} = \frac{1+1}{2} = \frac{3-5}{-2}$, (1, 1, 3) is a point on the first line as well. Hence, the lines are the same.

8. **a.** The line that passes through (0, 0, 3) with a directional vector of $(-3, 1, -6)$ is given by the parametric equation is $x = 3t, y = t,$ $z = 3 - 6t, t \in \mathbf{R}$. So, the y-coordinate is equal to -2 only when $t = -2$. At $t = -2, x = -3(-2) = 6$ and $z = 3 - 6(-2) = 15$. So, $A(6, -2, 15)$ is a point on the line. So, the y-coordinate is equal to 5 only when $t = 5$. At $t = 5$, $x = -3(5) = -15$ and $z = 3 - 6(5) = -27$. So, $B(-15, 5, -27)$ is a point on the line.

b. $x = -3t, y = t, z = 3 - 6t,$
$-2 \le t \le 5$

9. -1

10. **a.** $(8, 4, -3), (0, -8, 13), (4, -2, 5)$

b. $(-9, 3, 15), (1, 1, 3), (-4, 2, 9)$

c. $(-4, 3, -4), (2, 1, 4), (-1, 2, 0)$

d. $(-4, -1, -2), (-4, 5, 8) (-4, 2, 3)$

11. **a.** $x = 4 - 4t, y = -2 - 6t,$
$z = 5 + 8t, t \in \mathbf{R}$;
$\frac{x-4}{-4} = \frac{y+2}{-6} = \frac{z-5}{8}$

b. $\vec{r} = (-4, 2, 9) + s(5, -1, -6),$
$s \in \mathbf{R}; \frac{x+4}{5} = \frac{y-2}{-1} = \frac{z-9}{-6}$

c. $\vec{r} = (-1, 2, 0) + t(3, -1, 4), t \in \mathbf{R}$;
$x = -1 + 3t, y = 2 - t, z = 4t,$
$t \in \mathbf{R}$

d. $\vec{r} = (-4, 2, 3) + t(0, 3, 5), t \in \mathbf{R}$;
$x = -4, y = 2 + 3t, z = 3 + 5t,$
$t \in \mathbf{R}$

12. $x = 2 - 34t, y = -5 + 25t, z = 13t,$
$t \in \mathbf{R}$

13. $(-2, -1, 2), (2, 1, 2)$.

14. $P_1(2, 3, -2)$ and $P_2(4, -3, -4)$

15. about 17°

Mid-Chapter Review, pp. 451–452

1. **a.** $(-7, -2), (-5, 1), (-3, 4)$

b. $(-1, 5), (2, 3), (5, 1)$

c. $\left(-1, \frac{11}{5}\right), \left(0, \frac{8}{5}\right), (1, 1)$

d. $(-2, -4, 4), (4, 0, 6), (1, -2, 5)$

2. **a.** $\left(\frac{18}{5}, 0\right)$; $(0, 6)$

b. $\left(-3, 0\right)$; $(0, -3)$

3. approximately 86.8°

4. x-axis: about 51°; y-axis: about 39°

5. $5x - 7y - 41 = 0$

6. $\frac{x}{3} = \frac{y}{-4} = \frac{z-2}{4}$

7. $x = 1 + t, y = 2 - 9t, z = 5 + t, t \in \mathbf{R}$

8. approximately 79.3°, 137.7°, and 49.7°

9. $y = -4, \frac{x-3}{1} = \frac{z-6}{\sqrt{3}}$

10. x-axis: $x = t, y = 0, z = 0, t \in \mathbf{R}$;
y-axis: $x = 0, y = t, z = 0, t \in \mathbf{R}$;
z-axis: $x = 0, y = 0, z = t, t \in \mathbf{R}$

11. **a.** -7

b. $\frac{1}{19}$

12. 17.2 units, 12

13. **a.** $\vec{r} = (0, 6) + t(4, -3), t \in \mathbf{R}$

b. $x = 4t, y = 6 - 3t, t \in \mathbf{R}$

c. about 36.9°

d. $\vec{r} = t(3, 4), t \in \mathbf{R}$

14. $x + 6y - 32 = 0$;
$\vec{r} = (-4, 6) + t(12, -2), t \in \mathbf{R}$;
$x = -4 + 12t, y = 6 - 2t, t \in \mathbf{R}$

15. $\left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$

16. **a.** $x = -5 + 3t, y = 10 - 2t, t \in \mathbf{R}$

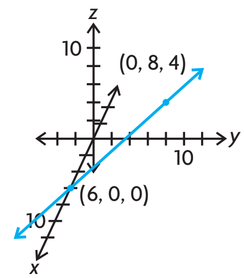
b. $x = 1 + t, y = -1 + t, t \in \mathbf{R}$

c. $x = 0, y = t, t \in \mathbf{R}$

17. **a.** yz-plane at (0, 8, 4); xz-plane at (6, 0, 0); xy-plane at (6, 0, 0)

b. x-axis at (6, 0, 0)

c.



18. **a.** $\vec{r} = (1, -2, 8) + t(-5, -2, 1),$
$t \in \mathbf{R}; x = 1 - 5t, y = -2 - 2t,$
$z = 8 + t, t \in \mathbf{R}$;
$\frac{x-1}{-5} = \frac{y+2}{-2} = \frac{z-8}{1}$

b. $\vec{r} = (3, 6, 9) + t(2, 4, 6), t \in \mathbf{R}$;
$x = 3 + 2t, y = 6 + 4t,$
$z = 9 + 6t, t \in \mathbf{R}$;
$\frac{x-3}{2} = \frac{y-6}{4} = \frac{z-9}{6}$

c. $\vec{r} = (0, 0, 6) + t(-1, 5, 1), t \in \mathbf{R}$;
$x = -t, y = 5t, z = 6 + t, t \in \mathbf{R}$;
$\frac{x}{-1} = \frac{y}{5} = \frac{z-6}{1}$

d. $\vec{r} = (2, 0, 0) + t(0, 0, -2), t \in \mathbf{R}$;
$x = 2, y = 0, z = -2t, t \in \mathbf{R}$; There is no symmetric equation for this line.

19. $\vec{r} = t(5, -5, -1), t \in \mathbf{R}$

20. $x = t, y = -8 - 13t, z = 1, t \in \mathbf{R}$

21. $(1, 3, -5), -3(1, 3, -5)$.

22. Since $\frac{7-4}{3} = \frac{-1+2}{1} = \frac{8-6}{2} = 1$, the point $(7, -1, 8)$ lies on the line.

Section 8.4 pp. 459–460

1. **a.** plane; This is a vector equation of a plane in $\mathbf{R}^3$.

b. line; This is a vector equation of a line in $\mathbf{R}^3$.

c. line; This is a parametric equation for a line in $\mathbf{R}^3$.

d. plane; This is a parametric equation of a plane in $\mathbf{R}^3$ using (0, 0, 0) as $\vec{r_0}$.

2. **a.** $(4, -24, 9)$

b. $(1, -2, 5)$

c. $\vec{r} = (2, 1, 3) + s(4, -24, 9)$
$+ t(1, -2, 5), t, s \in \mathbf{R}$

3. **a.** $(0, 0, -1)$

b. $(2, -3, -3)$ and $(0, 5, -2)$

c. $(-2, -17, 10)$

d. $m = 0$ and $n = 3$

e. For the point $B(0, 15, -8)$, the first two parametric equations are the same, yielding $m = 0$ and $n = 3$; however, the third equation would then give:
$-8 = -1 - 3m - 2n$
$-8 = -1 - 3(0) - 2(3)$
$-8 = -7$
which is not true. So, there can be no solution.

4. a. $\vec{r} = (-2, 3, 1) + t(0, 0, 1) + s(3, -3, 0), t, s \in \mathbf{R}$
b. $\vec{r} = (-2, 3, 2) + t(0, 0, 1) + s(3, -3, -1), t, s \in \mathbf{R}$

5. a. $\vec{r} = (1, 0, -1) + s(2, 3, -4) + t(4, 6, -8), t, s \in \mathbf{R}$, does not represent a plane because the direction vectors are the same. We can rewrite the second direction vector as $(2)(2, 3, -4)$. And so we can rewrite the equation as:
$\vec{r} = (1, 0, -1) + s(2, 3, -4) + 2t(2, 3, -4)$
$= (1, 0, -1) + (s + 2t)(2, 3, -4)$
$= (1, 0, -1) + n(2, 3, 4), n \in \mathbf{R}$
This is an equation of a line in $\mathbf{R}^3$.

6. a. $\vec{r} = (-1, 2, 7) + t(4, 1, 0) + s(3, 4, -1), t, s \in \mathbf{R}$;
$x = -1 + 4t + 3s$,
$y = 2 + t + 4s$,
$z = 7 - s, t, s \in \mathbf{R}$
b. $\vec{r} = (1, 0, 0) + t(-1, 1, 0) + s(-1, 0, 1), t, s \in \mathbf{R}$;
$x = 1 - t - s$,
$y = t$,
$z = s, t, s \in \mathbf{R}$
c. $\vec{r} = (1, 1, 0) + t(3, 4, -6) + s(7, 1, 2), t, s \in \mathbf{R}$;
$x = 1 + 3t + 7s$,
$y = 1 + 4t + s$,
$z = -6t + 2s, t, s \in \mathbf{R}$

7. a. $s = 1$ and $t = 1$
b. $(0, 5, -4) = (2, 0, 1) + s(4, 2, -1) + t(-1, 1, 2)$ gives the following parametric equations:
$0 = 2 + 4s + t \Rightarrow t = 2 + 4s$
$5 = 2s + t$
$5 = 2s + (2 + 4s)$
$3 = 6s$
$\frac{1}{2} = s$
$t = 2 + 4\left(\frac{1}{4}\right)$
$t = 2 + 2 = 4$
The third equation then says:
$-4 = 1 - s + 2t$
$-4 = 1 - \frac{1}{2} + 2(4)$
$-4 = \frac{17}{2}$, which is a false statement. So, the point $A(0, 5, -4)$ is not on the plane.

8. a. $\vec{l} = (-3, 5, 6) + s(-1, 1, 2), s \in \mathbf{R}$;
$\vec{p} = (-3, 5, 6) + t(2, 1, -3), t \in \mathbf{R}$
b. $(-3, 5, 6)$

9. $(0, 0, 5)$

10. $\vec{r} = (2, 1, 3) + s(4, 1, 5) + t(3, -1, 2), t, s \in \mathbf{R}$

11. $\vec{r} = m(2, -1, 7) + n(-2, 2, 3), m, n \in \mathbf{R}$

12. a. $(1, 0, 0)$, $(0, 1, 0)$ and $(1, 1, 0)$, $(-1, 1, 0)$
b. $\vec{r} = s(1, 0, 0) + t(0, 1, 0), t, s \in \mathbf{R}$;
$x = s$,
$y = t$,
$z = 0, t, s \in \mathbf{R}$

13. a. $\vec{r} = s(-1, 2, 5) + t(3, -1, 7), t, s \in \mathbf{R}$
b. $\vec{r} = (-2, 2, 3) + s(-1, 2, 5) + t(3, -1, 7), t, s \in \mathbf{R}$
c. The planes are parallel since they have the same direction vectors.

14. $(-4, 7, 1) - (-3, 2, 4) = (-1, 5, -3)$,
$\frac{27}{13}(-3, 2, 4) - \frac{17}{13}(-4, 7, 1) = (-1, -5, 7)$

15. $\vec{r} = (0, 3, 0) + t(0, 3, 2), t \in \mathbf{R}$

16. The fact that the plane $\vec{r} = \overrightarrow{OP_0} + s\vec{a} + t\vec{b}$ contains both of the given lines is easily seen when letting $s = 0$ and $t = 0$, respectively.

Section 8.5 pp. 468–469

1. a. $\vec{n} = (A, B, C) = (1, -7, -18)$
b. In the Cartesian equation:
$Ax + By + Cz + D = 0$
If $D = 0$, the plane passes through the origin.
c. $(0, 0, 0)$, $(11, -1, 1)$, $(-11, 1, -1)$

2. a. $\vec{n} = (A, B, C) = (2, -5, 0)$
b. In the Cartesian equation: $D = 0$. So, the plane passes through the origin.
c. $(0, 0, 0)$, $(5, 2, 0)$ $(5, 2, 1)$

3. a. $\vec{n} = (A, B, C) = (1, 0, 0)$
b. In the Cartesian equation: $D = 0$. So, the plane passes through the origin.
c. $(0, 0, 0)$, $(0, 1, 0)$ $(0, 0, 1)$

4. a. $x + 5y - 7z = 0$
b. $-8x + 12y + 7z = 0$

5. *Method 1:* Let $A(x, y, z)$ be a point on the plane. Then,
$\overrightarrow{PA} = (x + 3, y - 3, z - 5)$ is a vector on the plane.
$\vec{n} \cdot \overrightarrow{PA} = 0$
$(x + 3) + 7(y - 3) + 5(z - 5) = 0$
$x + 7y + 5z - 43 = 0.$
Method 2: $\vec{n} = (1, 7, 5)$ so the Cartesian equation is
$x + 7y + 5z + D = 0$
We know the point $(-3, 3, 5)$ is on the plane and must satisfy the equation, so
$(-3) + 7(3) + 5(5) + D = 0$
$43 + D = 0$
$D = -43$
This also gives the equation:
$x + 7y + 5z - 43 = 0.$

6. a. $7x + 19y - 3z - 28 = 0$
b. $7x + 19y - 3z - 28 = 0$
c. There is only one simplified Cartesian equation that satisfies the given information, so the equations must be the same.

7. $7x + 17y - 13z - 24 = 0$

8. $20x + 9y + 7z - 47 = 0$

9. a. $\left(\frac{2}{3}, \frac{2}{3}, -\frac{1}{3}\right)$
b. $\left(\frac{4}{\sqrt{26}}, -\frac{3}{\sqrt{26}}, \frac{1}{\sqrt{26}}\right)$
c. $\left(\frac{3}{13}, -\frac{4}{13}, \frac{12}{13}\right)$

10. $9x - 15y - z + 11 = 0$

11. $2x - 4y - z + 6 = 0$

12. a. First determine their normal vectors, $\vec{n_1}$ and $\vec{n_2}$. Then the angle between the two planes can be determined from the formula:
$\cos\theta = \frac{\vec{n_1} \cdot \vec{n_2}}{|\vec{n_1}||\vec{n_2}|}$
b. 30°

13. a. 53.3°
b. $2x - 3y - z + 5 = 0$

14. a. 8
b. $-\frac{5}{2}$
c. No, the planes cannot ever be coincident. If they were, then they would also be parallel, so $k = 8$, and we would have the two equations:
$4x + 8y - 2z + 1 = 0.$
$2x + 4y - z + 4 = 0 \Rightarrow$
$4x + 8y - 2z + 8 = 0$. Here all of the coefficients are equal except for the D-values, which means that they don't coincide.

15. $3x + 5y - z - 18 = 0$

16. $-\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y + \sqrt{3}z = 0$

17. $8x - 2y - 16z - 5 = 0$

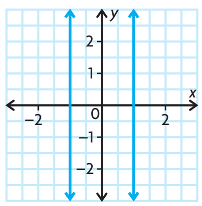
Section 8.6, pp. 476–477

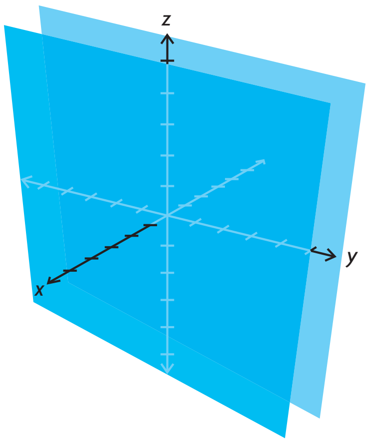
1. a. A plane parallel to the yz-axis, but two units away, in the negative x direction.
b. A plane parallel to the xz-axis, but three units away, in the positive y direction.
c. A plane parallel to the xy-axis, but 4 units away, in the positive z direction.

2. $(-2, 3, 4)$

3. P must lie on plane π_1 since the point has an x-coordinate of 5, and doesn't have a y-coordinate of 6.

4.



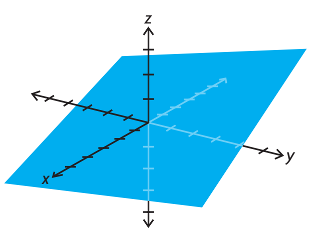


5. a. i. x-intercept $= 9$, y-intercept $= 6$, no z-intercept
ii. x-intercept $= 40$, y-intercept $= -30$, z-intercept $= 24$
iii. no x-intercept, y-intercept $= 3$, z-intercept $= -39$

b. i. $(0, 0, 1)$, $(3, -2, 0)$
ii. $(4, 3, 0)$, $(5, 0, -3)$
iii. $(1, 0, 0)$, $(0, 1, 13)$

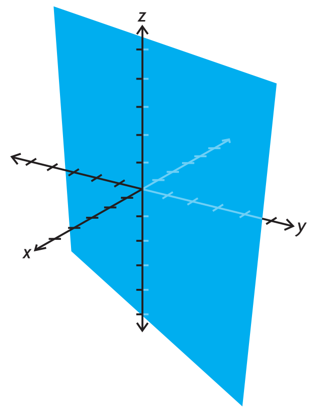
6. a. i. $(0, 0, 0)$, $(1, 2, 0)$, $(0, 5, 1)$
ii. $2x - y = 0$

b.

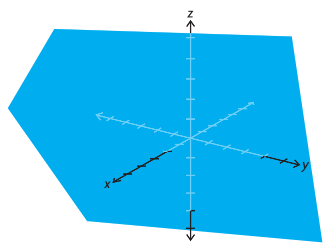


7. yz-plane, xz-plane, xy-plane

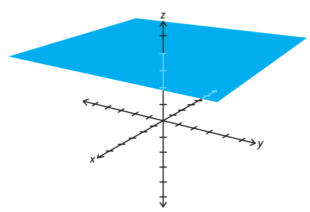
8. a.



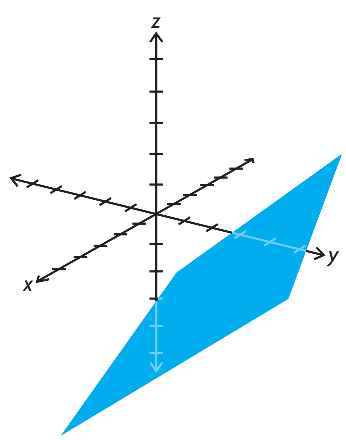
b.



c.

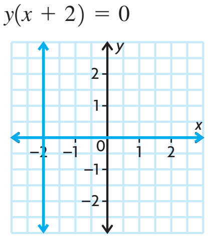


d.

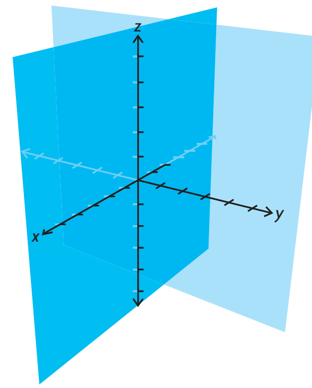


9. a. $y(x + 2) = 0$

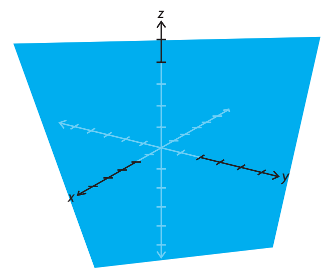
b.



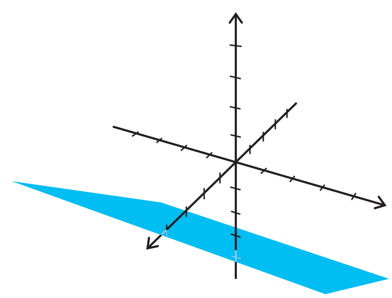
c.



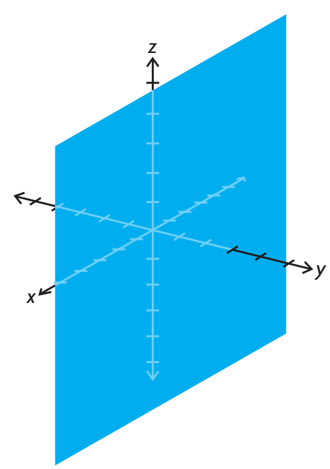
10. a.



b.



c.



11. a. $\frac{x}{3} + \frac{y}{4} + \frac{z}{6} = 1$

b. $\frac{x}{5} - \frac{z}{7} = 1$

c. $\frac{z}{8} = 1$

Review Exercise, pp. 480–483

1. Answers may vary. For example,
$\vec{r} = (1,2,-1) + s(1,-1,2) + t(1,0,3)$
$t \in \mathbf{R}$;
$x = 1 + s + t$,
$y = 2 - s$,
$z = -1 + 2s + 3t$

2. $3x + y - z - 6 = 0$
$\overrightarrow{AC} = (2, -1, 5) = \vec{c}$
$\overrightarrow{BC} = (1, 0, 3) = \vec{b}$
$\vec{r} = (1, 2, -1) + s(2, -1, 5)$
$+ t(1, 0, 3), s, t \in \mathbf{R}$
$\vec{b} \times \vec{c} = (1, 0, 3) \times (2, -1, 5)$
$= (3, 1, -1)$
$Ax + By + Cz + D = 0$
$(3)x + (1)y + (-1)z + D = 0$
$3(1) + (2) - 1(-1) + D = 0$
$D = -6$
$3x + y - z - 6 = 0$
Both Cartesian equations are the same regardless of which vectors are used.

3. **a.** Answers may vary. For example,
$\vec{r} = (4, 3, 9) + t(7, 1, 1), t \in \mathbf{R}$;
$x = 4 + 7t, y = 3 + t, z = 9 + t$,
$t \in \mathbf{R}$;
$\frac{x - 4}{7} = \frac{y - 3}{1} = \frac{z - 9}{1}$

b. Answers may vary. For example,
$\vec{r} = (4, 3, 9) + t(7, 1, 1)$
$+ s(3, 2, 3), t, s \in \mathbf{R}$;
$x = 4 + 7t + 3s, y = 3 + t + 2s$,
$z = 9 + t + 3s, t, s \in \mathbf{R}$

c. There are two parameters.

4. $\vec{r} = (7, 1, -2) + t(2, -3, 1), t \in \mathbf{R}$;
$x = 7 + 2t, y = 1 - 3t, z = -2 + t$;
$\frac{x - 7}{2} = \frac{y - 1}{-3} = \frac{z + 2}{1}$

5. **a.** $x - 3y - 3z - 3 = 0$
b. $3x + 5y - 2z - 7 = 0$
c. $3y + z - 7 = 0$

6. $19x - 7y - 8z = 0$

7. $\vec{r} = (-1, 2, 1) + t(0, 1, 0)$
$+ s(0, 0, 1)\ t, s \in \mathbf{R}$;
$x = -1, y = 2 + t, z = 1 + s$

8. $3x + y - z - 7 = 0$

9. $34x + 32y - 7z - 229 = 0$

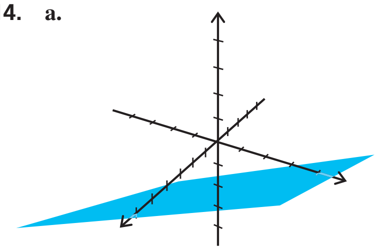
10. Answers may vary. For example,
$\vec{r} = (2, 3, -3) + s(3, -2, 1), s \in \mathbf{R}$;
$x = 2 + 3s, y = 3 - 2s, z = -3 + s$;
$\frac{x - 2}{3} = \frac{y - 3}{-2} = \frac{z + 3}{1}$

11. Answers may vary. For example,
$\vec{r} = (0, 0, 6) + s(1, 0, 3)$
$+ t(3, -5, -1), s, t \in \mathbf{R}$;
$x = s + 3t, y = -5t, z = 6 + 3s - t$

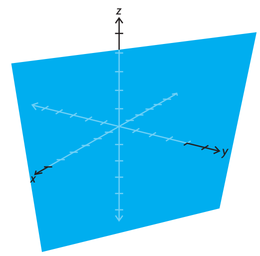
12. Answers may vary. For example,
$\vec{r} = (0, 0, 7) + t(1, 0, 2), t \in \mathbf{R}$;
$x = t, y = 0, z = 7 + 2t$;
$\frac{x}{1} = \frac{z - 7}{2}, y = 0$

13. $\vec{r} = (3, -4, 1) + s(1, -3, -5)$
$+ t(4, 3, -1), s, t \in \mathbf{R}$;
$x = 3 + s + 4t$,
$y = -4 - 3s + 3t$
$z = 1 - 5s - t, s, t \in \mathbf{R}$;
$18x - 19y + 15z - 145 = 0$

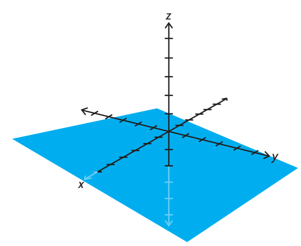
14. **a.**



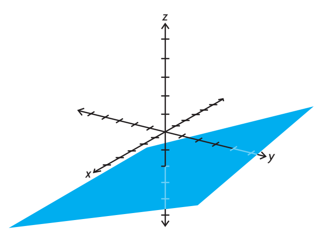
b.



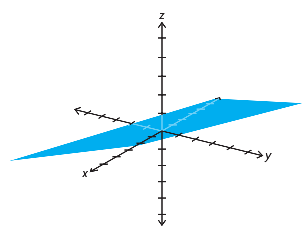
c.



d.



e.



15. **a.** Answers may vary. For example,
$\vec{r} = (3, 1, 2) + t(2, 4, 1)$
$+ s(2, 3, -3), t, s \in \mathbf{R}$;
$x = 3 + 2t + 2s$,
$y = 1 + 4t + 3s, z = 2 + t - 3s$;
$15x - 8y + 2z - 41 = 0$

b. Answers may vary. For example,
$\overrightarrow{BC} = (-4, 0, 11)$
$D = -18$
$\left(0, 0, \frac{18}{11}\right) + s\left(1, 0, \frac{4}{11}\right) +$
$t(0, 1, 0), s, t \in \mathbf{R}$
$-4x + 11z - 18 = 0$

c. Answers may vary. For example,
$\vec{r} = (4, 1, -1) + t(1, -3, 5)$
$+ s(0, 0, 1), t, s \in \mathbf{R}$;
$x = 4 + t, y = 1 - 3t$,
$z = -1 + 5t + s$;
$3x + y - 13 = 0$

d. Answers may vary. For example,
$\vec{r} = (1, 3, -5) + t(1, 3, 9)$
$+ s(1, -9, -1), t, s \in \mathbf{R}$;
$x = 1 + t + s, y = 3 + 3t - 9s$,
$z = -5 + 9t - s$;
$78x + 10y - 12z - 168 = 0$

16. They are in the same plane because both planes have the same normal vectors and Cartesian equations.
$L_1: \vec{r} = (1, 2, 3) + s(-3, 5, 21)$
$+ t(0, 1, 3), s, t \in \mathbf{R}$
$L_2: \vec{r} = (1, -1, -6) + u(1, 1, 1)$
$+ v(2, 5, 11), u, v \in \mathbf{R}$
$(-3, 5, 21) \times (0, 1, 3) = (-6, 9, -3)$
$= (2, -3, 1)$
$(1, 1, 1) \times (2, 5, 11) = (6, -9, 3)$
$= (2, -3, 1)$
$Ax + By + Cz + D = 0$
$2x - 3y + z + D = 0$
$2(1) - 3(2) + (3) + D = 0$
$D = 1$
$2(1) - 3(-1) + (-6) + D = 0$
$D = 1$
$2x - 3y + z + 1 = 0$

17. $\left(\frac{20}{3}, \frac{10}{3}, -\frac{1}{3}\right)$

18. **a.** The plane is parallel to the z-axis through the points (3, 0, 0) and (0, −2, 0).
b. The plane is parallel to the y-axis through the points (6, 0, 0) and (0, 0, −2).
c. The plane is parallel to the x-axis through the points (0, 3, 0) and (0, 0, −6).

19. **a.** A
b. $a = -8, b = -1$

20. **a.** 45.0° **b.** 59.0° **c.** 37.4° **d.** 90°

21. **a.** 17.0°
b. 90°

22. **a.** **i.** no **ii.** yes **iii.** no
b. **i.** yes **ii.** no **iii.** no

23. $(x, y, z) = (4, 1, 6) + p(3, -2, 1)$
$+ q(-6, 6, -1)$
$(x, y, z) = (4, 1, 6) + 4(3, -2, 1)$
$+ 2(-6, 6, -1)$
$(x, y, z) = (4, 5, 8) \neq (4, 5, 6)$

24. $x = 1 + s + 3t, y = 4 - t$,
$z = 4 - 3s + t, s, t \in \mathbf{R}$

25. A plane has two parameters, because a plane goes in two different directions, unlike a line that goes only in one direction.

26. This equation will always pass through the origin, because you can always set $s = 0$ and $t = -1$ to obtain (0, 0, 0).

27. a. They do not form a plane, because these three points are collinear.
$\vec{r} = (-1, 2, 1) + t(3, 1, -2)$
b. They do not form a plane, because the point lies on the line.
$\vec{r} = (4, 9, -3) + t(1, -4, 2)$
$\vec{r} = (4, 9, -3) + 4(1, -4, 2)$
$= (8, -7, 5)$

28. $bcx + acy + abz - abc = 0$

29. $6x - 5y + 12z + 46 = 0$

30. a., b. $\vec{r} = (1, -3, 2) + t(-3, 7, -4) + s(5, -2, 3)\ t, s \in \mathbf{R}$;
$x = 1 - 3t + 5s,$
$y = -3 + 7t - 2s,$
$z = 2 - 4t + 3s$
c. $13x - 11y - 29z + 12 = 0$
d. no

31. a. $4x - 2y + 5z = 0$
b. $4x - 2y + 5z + 19 = 0$
c. $4x - 2y + 5z - 22 = 0$

32. a. These lines are coincident. The angle between them is 0°.
b. $\left(\frac{3}{2}, 5\right)$, 86.82°

33. a. $\vec{r} = (1, 3, 5) + t(-2, -4, -10)$, $t \in \mathbf{R}$;
$x = 1 - 2t, y = 3 - 4t,$
$z = 5 - 10t;$
$\frac{x - 1}{-2} = \frac{y - 3}{-4} = \frac{z - 5}{-10}$
b. $\vec{r} = (1, 3, 5) + t(-8, 6, -2), t \in \mathbf{R}$;
$x = 1 - 8t, y = 3 + 6t,$
$z = 5 - 2t;$
$\frac{x - 1}{-8} = \frac{y - 3}{6} = \frac{z - 5}{-2}$
c. $\vec{r} = (1, 3, 5) + t(-6, -13, 14)$, $t \in \mathbf{R}$;
$x = 1 - 6t, y = 3 - 13t,$
$z = 5 + 14t;$
$\frac{x - 1}{-6} = \frac{x - 3}{-13} = \frac{x - 5}{14}$
d. $\vec{r} = (1, 3, 5) + t(1, 0, 0), t \in \mathbf{R}$;
$x = 1 + t, y = 3, z = 5$
e. $x = 1, y = 3 + 6t, z = 5 + 4t;$ $t \in \mathbf{R}$
$\vec{r} = (1, 3, 5) + t(0, 6, 4), t \in \mathbf{R}$
f. $\vec{r} = (1, 3, 5) + t(0, 1, 6)$;
$x = 1, y = 3 + t, z = 5 + 6t$

34. a. $2x - 4y + 5z + 23 = 0$
b. $29x + 27y + 24z - 86 = 0$
c. $z - 3 = 0$
d. $3x + y - 4z + 26 = 0$
e. $y - 2z - 4 = 0$
f. $-5x + y + 7z + 18 = 0$

Chapter 8 Test, p. 484

1. a. i. $\vec{r} = (1, 2, 4) + s(1, -2, -1) + t(3, 2, 0), s, t \in \mathbf{R}$;
$x = 1 + s + 3t,$
$y = 2 - 2s + 2t, z = 4 - s,$
$s, t \in \mathbf{R}$
ii. $2x - 3y + 8z - 28 = 0$
b. no

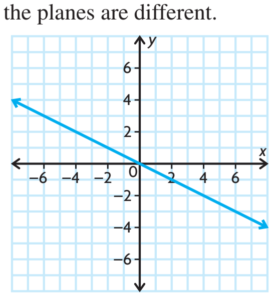
2. a. $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$
b. (6, 4, 3)

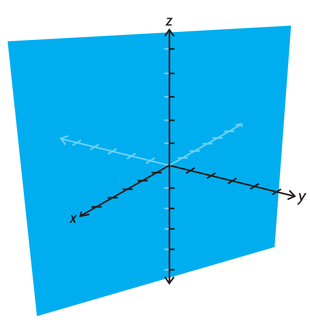
3. a. $\vec{r} = s(2, 1, 3) + t(1, 2, 5), s, t \in \mathbf{R}$
b. $-x - 7y + 3z = 0$

4. a. $\vec{r} = (4, -3, 5) + s(2, 0, -3) + t(5, 1, -1), s, t \in \mathbf{R}$
b. $3x - 13y + 2z - 61 = 0$

5. a. $\left(0, 5, -\frac{1}{2}\right)$
b. $\frac{x}{4} = \frac{y - 5}{-2} = \frac{1}{2}$

6. a. about 70.5°
b. i. 4
ii. $-\frac{1}{5}$
c. The y-intercepts are different and the planes are different.

7. a.


b.


c. The equation for the plane can be written as $Ax + By + 0z = 0$. For any real number t, $A(0) + B(0) + 0(t) = 0$, so $(0, 0, t)$ is on the plane. Since this is true for all real numbers, the z-axis is on the plane.

Chapter 9

Review of Prerequisite Skills, p. 487

1. a. yes
b. no
c. yes
d. no

2. Answers may vary. For example:
a. $\vec{r} = (2, 5) + t(5, -2), t \in \mathbf{R}$;
$x = 2 + 5t, y = 5 - 2t, t \in \mathbf{R}$
b. $\vec{r} = (-3, 7) + t(7, -14), t \in \mathbf{R}$;
$x = -3 + 7t, y = 7 - 14t, t \in \mathbf{R}$
c. $\vec{r} = (-1, 0) + t(-2, -11), t \in \mathbf{R}$;
$x = -1 - 2t, y = -11t, t \in \mathbf{R}$
d. $\vec{r} = (1, 3, 5) + t(5, -10, -5), t \in \mathbf{R}$;
$x = 1 + 5t, y = 3 - 10t, z = 5 - 5t,$ $t \in \mathbf{R}$
e. $\vec{r} = (2, 0, -1) + t(-3, 5, 3), t \in \mathbf{R}$;
$x = 2 - 3t, y = 5t, z = -1 + 3t,$ $t \in \mathbf{R}$
f. $\vec{r} = (2, 5, -1) + t(10, -10, -6)$, $t \in \mathbf{R}$;
$x = 2 + 10t, y = 5 - 10t, z = -1 - 6t, t \in \mathbf{R}$

3. a. $2x + 6y - z - 17 = 0$
b. $y = 0$
c. $4x - 3y - 15 = 0$
d. $6x - 5y + 3z = 0$
e. $11x - 6y - 38 = 0$
f. $x + y - z - 6 = 0$

4. $5x + 11y + 2z - 21 = 0$

5. L_1 is not parallel to the plane. L_1 is on the plane.
L_2 is parallel to the plane.
L_3 is not parallel to the plane.

6. a. $x - y - z - 2 = 0$
b. $x + 6y - 10z - 30 = 0$

7. $\vec{r} = (1, -4, 3) + t(1, 3, 3) + s(0, 1, 0), s, t \in \mathbf{R}$
$-3x + z = 0$

8. $3y + z = 13$

Section 9.1, pp. 496–498

1. a. $\pi: x - 2y - 3z = 6,$
$\vec{r} = (1, 2, -3) + s(5, 1, 1)\ s \in \mathbf{R}$
b. This line lies on the plane.

2. a. A line and a plane can intersect in three ways: (1) The line and the plane have zero points of intersection. This occurs when the lines are not incidental, meaning they do not intersect.
(2) The line and the plane have only one point of intersection. This occurs when the line crosses the plane at a single point.
(3) The line and the plane have an infinite number of intersections. This occurs when the line is

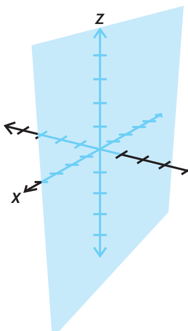
coincident with the plane, meaning the line lies on the plane.

b. Assume that the line and the plane have more than one intersection, but not an infinite number. For simplicity, assume two intersections. At the first intersection, the line crosses the plane. In order for the line to continue on, it must have the same direction vector. If the line has already crossed the plane, then it continues to move away from the plane, and can not intersect again. So, the line and the plane can only intersect zero, one, or infinitely many times.

3. a. The line $\vec{r} = s(1, 0, 0)$ is the x-axis.

b. The plane is parallel to the xz-plane, but just one unit away to the right.

c.



d. There are no intersections between the line and the plane.

4. a. For $x + 4y + z - 4 = 0$, if we substitute our parametric equations, we have $(-2 + t) + 4(1 - t) + (2 + 3t) - 4 = 0$
All values of t give a solution to the equation, so all points on the line are also on the plane.

b. For the plane $2x - 3y + 4z - 11 = 0$, we can substitute the parametric equations derived from $\vec{r} = (1, 5, 6) + t(1, -2, -2)$:
$2(1 + t) - 3(5 - 2t) + 4(6 - 2t) - 11 = 0$
All values of t give a solution to this equation, so all points on the line are also on the plane.

5. a. $2(-1 - s) - 2(1 + 2s) + 3(2s) - 1 = -5$
Since there are no values of s such that $-5 = 0$, this line and plane do not intersect.

b. $2(1 + 2t) - 4(-2 + 5t) + 4(1 + 4t) - 13 = 1$
Since there are no values of t such that $1 = 0$, there are no solutions, and the plane and the line do not intersect.

6. a. The direction vector is $\vec{m} = (-1, 2, 2)$ and the normal is $\vec{n} = (2, -2, 3)$, $\vec{m} \cdot \vec{n} = 0$. So the line is parallel to the plane, but $2(-1) - 2(1) + 3(0) - 1 = -5 \neq 0$. So, the point on the line is not on the plane.

b. The direction vector is $\vec{m} = (2, 5, 4)$ and the normal is $\vec{n} = (2, -4, 4)$, $\vec{m} \cdot \vec{n}$, $= 0$, so the line is parallel to the plane. and
$2(1) - 4(-2) + 4(1) - 13 = 1 \neq 0$
So, the point on the line is not on the plane.

7. a. $(-19, 0, 10)$

b. $(-11, 1, 0)$

8. a. There is no intersection and the lines are skew.

b. $(4, 1, 2)$

9. a. not skew

b. not skew

c. not skew

d. skew

10. 8

11. a. Comparing components results in the equation $s - t = -4$ for each component.

b. From L_1, we see that at $(-2, 3, 4)$, $s = 0$. When this occurs, $t = 4$. Substituting this into L_2, we get $(-30, 11, -4) + 4(7, -2, 2) = (-2, 3, 4)$. Since both of these lines have the same direction vector and a common point, the lines are coincidental.

12. a. 3

b. $\left(\frac{2}{11}, \frac{53}{11}, \frac{46}{11}\right)$

13. 3

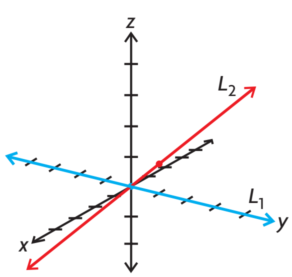
14. a. $(-6, 1, 3)$

b. 3

15. a. $(4, 1, 12)$

b. $\vec{r} = (4, 1, 12) + t(42, 55, -10)$, $t \in \mathbf{R}$

16. a.



b. $(0, 0, 0)$

c. If $p = 0$ and $q = 0$, the intersection occurs at $(0, 0, 0)$.

17. a. Represent the lines parametrically, and then substitute into the equation for the plane. For the first equation, $x = t, y = 7 - 8t, z = 1 + 2t$.
Substituting into the plane equation, $2t + 7 - 8t + 3 + 6t - 10 = 0$.
Simplifying, $0t = 0$. So, the line lies on the plane.
For the second line, $x = 4 + 3s, y = -1, z = 1 - 2s$.
Substituting into the plane equation, $8 + 6s - 1 + 3 - 6s - 10 = 0$.
Simplifying, $0s = 0$. This line also lies on the plane.

b. $(1, -1, 3)$

18. Answers may vary. For example, $\vec{r} = (2, 0, 0) + p(2, 0, 1)$, $p \in \mathbf{R}$.

Section 9.2, pp. 507–509

1. a. linear

b. not linear

c. linear

d. not linear

2. Answers may vary. For example:

a. $x + y + 2z = -15$
$x + 2y + z = -3$
$2x + y + z = -10$

b. $(-3, 4, -8)$

3. a. yes

b. no

4. a. $(-2, -3)$

b. $(-2, -3)$
The two systems are equivalent because they have the same solution.

5. a. $(6, 1)$

b. $(-3, 5)$

c. $(-4, 3)$

6. a. These two lines are parallel, and therefore cannot have an intersection.

b. The second equation is five times the first; therefore, the lines are coincident.

7. a. $x = t, y = 2t - 3, t \in \mathbf{R}$

b. $x = t, y = s, z = 2s - t, t \in \mathbf{R}$

8. a. $2x + y = -11$

b. $2x + y = -11$
$2(3t + 3) + (-6t - 17) =$
$6t - 6t + 6 - 17 = -11$

9. a. $k \neq 12$

b. not possible

c. $k = 12$

10. **a** infinitely many

b. $x = t$,

$y = \frac{11}{4} - \frac{1}{2}t, t \in \mathbf{R}$

c. This equation will not have any integer solutions because the left side is an even function and the right side is an odd function.

11. **a.** $x = -a + b, y = -\frac{1}{3}b + \frac{2}{3}a$

b. Since they have different direction vectors, these two equations are not parallel or coincident and will intersect somewhere.

12. **a.** $(-1, -2, 3)$

b. $(3, 4, 12)$

c. $(4, 6, -8)$

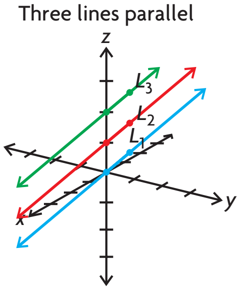
d. $(60, 120, -180)$

e. $(2, 4, 1)$

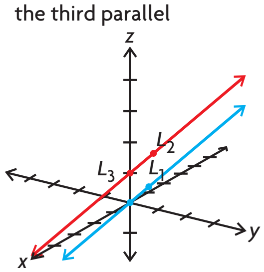
f. $(-2, 3, 6)$

13. Answers may vary. For example:

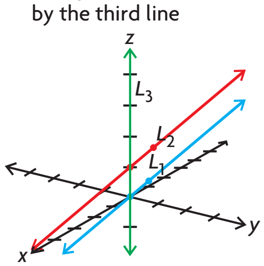
a. Three lines parallel



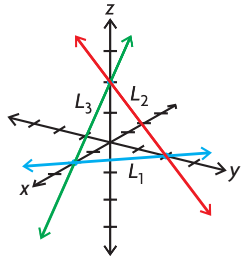
Two lines coincident and the third parallel



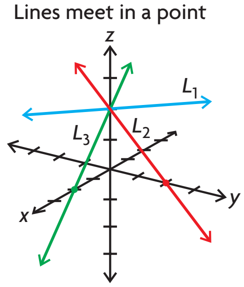
Two parallel lines cut by the third line



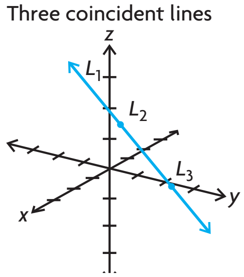
The lines form a triangle



b. Lines meet in a point



c. Three coincident lines



14. $(a - c, -a + b + c, a - b)$

15. **a.** $k = 2$

b. $k = -2$

c. $k \neq \pm 2$

Section 9.3, pp. 516–517

1. **a.** The two equations represent planes that are parallel and not coincident.

b. Answers may vary. For example: $x - y + z = 1, x - y + z = -2$

2. **a.** $x = \frac{1}{2} + \frac{1}{2}s - t, y = s, z = t;$ $s, t \in \mathbf{R}$; the two planes are coincident.

b. Answers may vary. For example: $x - y + z = -1$, $2x - 2y + 2z = -2$

3. **a.** $x = 1 + s, y = s, z = -2, s \in \mathbf{R}$; the two planes intersect in a line.

b. Answers may vary. For example: $x - y + z = -1, x - y - z = 3$

4. **a.** $m = \frac{1}{2}, p = 2q, q = 1$, and $p = 2$; The value for m is unique, but p just has to be twice q and arbitrary values can be chosen.

b. $m = \frac{1}{2}, q = 1$, and $p = 3$; The value for m is unique, but p and q can be arbitrarily chosen as long as $p \neq 2q$.

c. $m = -20$; This value is unique, since only one value was found to satisfy the given conditions.

d. $m = -20, p = 1, q = 1$; The value for m is unique from the solution to **c.**, but the values for p and q can be arbitrary since the only value which can change the angle between the planes is m.

5. **a.** $x = 9s, y = -3s, z = s, s \in \mathbf{R}$

b. $x = -3t, y = t, z = -\frac{1}{3}t, t \in \mathbf{R}$

c. Since t is an arbitrary real number, we can express t as part b. $t = -3s$, $s \in \mathbf{R}$.

6. **a.** yes; plane

b. no

c. yes; line

d. yes; line

e. yes; line

f. yes; line

7. **a.** $x = 1 - s - t, y = s, z = t, s, t \in \mathbf{R}$

b. no solution

c. $x = -2s, y = -2, z = s, s \in \mathbf{R}$

d. $x = -s + 5, y = -s - 1, z = s$, $s \in \mathbf{R}$

e. $x = \frac{5}{4}s, y = s, z = 1 - \frac{3}{4}s, s \in \mathbf{R}$

f. $x = s - 8, y = s, z = 4, s \in \mathbf{R}$

8. **a.** The system will have an infinite number of solutions for any value of k.

b. No, there is no value of k for which the system will not have a solution.

9. $\vec{r}_2 = (-2, 3, 6) + s(-5, -8, 2)$, $s \in \mathbf{R}$

10. The line of intersection of the two planes,

$x = 1 - 2s, y = 2 - 2s, z = s; s \in \mathbf{R}$;

$5x + 3y + 16z - 11 = 0$

$5(1 - 2s) + 3(2 - 2s) +$

$16(s) - 11 = 0$

$5 + 6 - 11 - 10s - 6s + 16s = 0$

$0 = 0$

Since this is true, the line is contained in the plane.

11. **a.** $x = 1 + s, y = 1 + s, z = s, s \in \mathbf{R}$

b. about 1.73

12. $8x + 14y - 3z - 8 = 0$

Mid-Chapter Review, pp. 518–519

1. **a.** $(-2, 6, 0)$
b. $(2, 0, 10)$
c. $(0, 3, 5)$

2. **a, c.** Answers may vary. For example:
$x = 2 + 3t, y = 1 + 3t,$
$z = 3 - 2t, t \in \mathbf{R};$
$x = 3 + t, y = -2, z = 5\ t \in \mathbf{R};$
$x = -8 + 7t, y = -5 + 3t,$
$z = 7 - 2t, t \in \mathbf{R}$
b. $(-1, -2, 5)$
d. $C: x = -8 + 7t, y = -5 + 3t,$
$z = 7 - 2t, t \in \mathbf{R}$
$t = 1$
$x = -8 + 7(1), y = -5 + 3(1),$
$z = 7 - 2(1)$
$x = -1, y = -2, z = 5$
$(-1, -2, 5)$
e. $(-1, -2, 5)$

3. **a.** $\vec{r} = (-7, 20, 0) + t(0, -2, 1),$
$t \in \mathbf{R}$
b. $\vec{r} = \left(-\frac{19}{7}, \frac{30}{7}, 0\right) + t(3, 3, -7),$
$t \in \mathbf{R}$
c. $(-7, 0, 10)$

4. **a.** $x = -\frac{11t}{5} - \frac{1}{40}, y = -\frac{2t}{5} - \frac{117}{40},$
$z = t, t \in \mathbf{R}$
b. $x = -\frac{11}{5}s + \frac{227}{5}, y = -\frac{2}{5}s + \frac{94}{5},$
$z = s, t \in \mathbf{R}$
c. The lines found in 4.a. and 4.b. do not intersect, because they are in parallel and distinct planes.

5. **a.** $a = 3$
b. $a = -3$
c. $a \neq \pm 3, a \in \mathbf{R}$

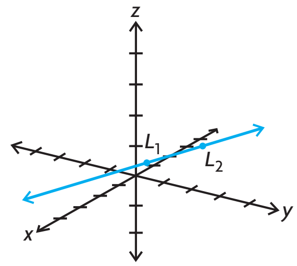
6. Since there is no t-value that satisfies the equations, there is no intersection, and these lines are skew.

7. **a.** no intersection
b. The lines are skew.

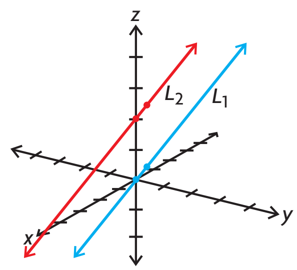
8. $(-3, 6, 6)$

9. **a.** $(3, 1, 2)$
b. These lines are the same, so either one of these lines can be used as their intersection.

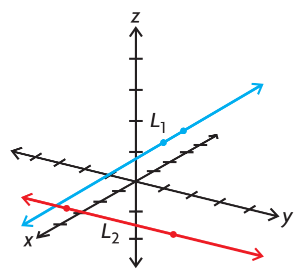
10. **a.** Answers may vary. For example:
i. coincident



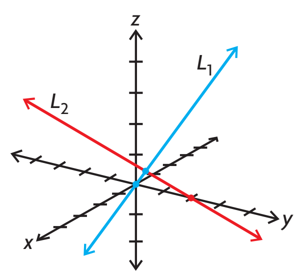
ii. parallel and distinct



iii. skew



iv. intersect in a point



b. **i.** When lines are the same, they are a multiple of each other.
ii. When lines are parallel, one equation is a multiple of the other equation, except for the constant term.
iii. When lines are skew, there are no common solutions to make each equation consistent.
iv. When the solution meets in a point, there is only one unique solution for the system.

11. **a.** when the line lies in the plane
b. Answers may vary. For example:
$\vec{r} = t(3, -5, -3), t \in \mathbf{R};$
$\vec{r} = t(3, -5, -3) + s(1, 1, 1),$
$t, s \in \mathbf{R}$

12. **a.** $(3, 8)$
b. no solution
c. $(2, 1, 4)$

13. **a.** The two lines intersect at a point.
b. The two planes are parallel and do not meet.

14. **a.** $\left(-\frac{1}{2}, -\frac{3}{2}, -\frac{3}{2}\right)$
b. $\theta = 90°$
c. $2x - y + z + 1 = 0$

Section 9.4, pp. 530–533

1. **a.** $(-9, -5, -4)$
b. This solution is the point at which all three planes meet.

2. **a.** Answers may vary. For example,
$3x - 3y + 3z = 12$ and
$2x - 2y + 2z = 8$
$x - y + z = 4.$
b. These three planes are intersecting in one single plane because all three equations can be changed into one equivalent equation. They are coincident planes.
c. $x = t, y = s, z = s - t + 4, s, t \in \mathbf{R}$
d. $y = t, z = s, x = t - s + 4, s, t \in \mathbf{R}$

3. **a.** Answers may vary. For example,
$2x - y + 3z = -2, x - y + 4z = 3,$
and $3x - 2y + 7z = 2;$
$2x - y + 3z = -2, x - y + 4z = 3,$
and $2x - 2y + 8z = 5.$
b. no solutions

4. **a.** $\left(-3, \frac{11}{4}, -\frac{3}{2}\right)$
b. This solution is the point at which all three planes meet.

5. **a.** Since equation ③ = − 3 × equation ②, equation ② and equation ③ are consistent or lie in the same plane. Equation ① meets this plane in a line.
b. $x = 0, y = t,$ and $z = 1 + t, t \in \mathbf{R}$

6. If you multiply equation ② by 5, you obtain a new equation, $5x - 5y + 15z = -1005$, which is inconsistent with equation ③.

7. **a.** Yes, when this equation is alone, this is true.
b. Answers may vary. For example:
$x + y + z = 2$
$2x + 2y + 2z = 4$
$3x + 3y + 3z = 12$

8. a. $(-1, -1, 0)$ is the point at which the three planes meet.
b. $(-6, \frac{1}{2}, 3)$ is the point at which the three planes meet.
c. $(-99, 100, -101)$ is the point at which the three planes meet.
d. $(4, 2, 3)$ is the point at which the three planes meet.

9. a. $x = -\frac{1}{7}t - \frac{9}{7}, y = -\frac{15}{7} + \frac{3}{7}t$, and $z = t, t \in \mathbf{R}$; the planes intersect in a line.
b. no solution
c. $x = -t, y = 2$, and $z = t, t \in \mathbf{R}$; the planes intersect in a line.

10. a. $x = 0, y = t - 2$, and $z = t, t \in \mathbf{R}$
b. $x = \frac{t - 3s}{2}, y = t$, and $z = s$, $s, t \in \mathbf{R}$

11. a. ① $x + y + z = 1$
② $x - 2y + z = 0$
③ $x - y + z = 0$
Equation ① − equation ③ =
Equation ④ $= 2y = 1$ or $y = \frac{1}{2}$
Equation ② − equation ③ =
Equation ⑤ $= -y = 0$ or $y = 0$
Since the y-variable is different in Equation ④ and Equation ⑤, the system is inconsistent and has no solution.
b. Answers may vary. For example:
$\vec{n_1} = (1, 1, 1)$
$\vec{n_2} = (1, -2, 1)$
$\vec{n_3} = (1, -1, 1)$
$m_1 = \vec{n_1} \times \vec{n_2} = (3, 0, -3)$
$m_2 = \vec{n_1} \times \vec{n_3} = (2, 0, -2)$
$m_3 = \vec{n_2} \times \vec{n_3} = (-1, 0, 1)$
c. The three lines of intersection are parallel and coplanar, so they form a triangular prism.
d. Since $(\vec{n_1} \times \vec{n_2}) \cdot \vec{n_3} = 0$, a triangular prism forms.

12. a. Equation ① and equation ② have the same set of coefficients and variables; however, equation ① equals 3, while equation ② equals 6, which means there is no possible solution.
b. All three equations equal different numbers, so there is no possible solution.
c. Equation ② equals 18, while equation ③ equals 17, which means there is no possible solution.
d. The coefficients of equation ① are half the coefficients of equation ③, but the constant term is not half the other constant term.

13. a. $(4, 3, -5)$
b. $x = \frac{t - 2}{3}, y = \frac{5t + 5}{3}, z = t, t \in \mathbf{R}$
c. $x = 0, y = t, z = t, t \in \mathbf{R}$
d. no solution
e. $x = -t, y = 2, z = t, t \in \mathbf{R}$
f. $(0, 0, 0)$

14. a. $p = q = 5$
b. $x = -\frac{2}{3}t + 3, y = \frac{1}{3}t - 2, z = t$, $t \in \mathbf{R}$

15. a. $m = 2$
b. $m \neq \pm 2, m \in \mathbf{R}$
c. $m = -2$

16. $(3, 6, 2)$

Section 9.5, pp. 540–541

1. a. $\frac{3}{5}$
b. $\frac{56}{13}$ or 4.31
c. $\frac{236}{\sqrt{1681}}$ or 5.76

2. a. $\frac{5}{\sqrt{5}}$ or 2.24
b. $\frac{504}{25}$ or 20.16

3. a. 1.4
b. about 3.92
c. about 2.88

4. a. $d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$
If you substitute the coordinates $(0, 0)$, the formula changes to $d = \frac{|A(0) + B(0) + C|}{\sqrt{A^2 + B^2}}$, which reduces to $d = \frac{|C|}{\sqrt{A^2 + B^2}}$.
b. $\frac{24}{5}$
c. $\frac{24}{5}$; the answers are the same

5. a. 3
b. $\frac{7}{5}$ or 1.4
c. $\frac{4}{\sqrt{13}}$ or 1.11
d. $\frac{240}{13}$ or 18.46

6. a. about 1.80
b. about 2.83
c. about 3.44

7. a. about 2.83
b. about 3.81

8. a. $\left(\frac{17}{11}, \frac{7}{11}, \frac{16}{11}\right)$
b. about 1.65

9. about 3.06; $\left(-\frac{11}{14}, \frac{5}{14}, \frac{22}{14}\right)$

10. $\left(\frac{38}{21}, -\frac{44}{21}, \frac{167}{21}\right)$

11. a. about 1.75
b. D and G
c. about 3.61 units^2

Section 9.6, pp. 549–550

1. a. Yes, the calculations are correct. Point A lies in the plane.
b. The answer 0 means that the point lies in the plane.

2. a. 3 b. 3 c. 2 d. $\frac{5}{13}$ or 0.38 e. $\frac{11}{27}$ or 0.41

3. a. 5
b. $6x + 8y - 24z + 13 = 0$
c. Answers may vary. For example: $\left(-\frac{1}{6}, 0, \frac{1}{2}\right)$

4. a. 4 b. 4 c. 2

5. $\frac{2}{3}$ or 0.67

6. 3

7. about 1.51

8. a. about 3.46
b. $U(1, 1, 2)$ is the point on the first line that produces the minimal distance to the second line at point $V(-1, -1, 0)$.

Review Exercise, pp. 552–555

1. $-\frac{4}{99}$

2. no solution

3. a. no solution
b. $(99, 100, 101)$

4. a. All four points lie on the plane $3x + 4y - 2z + 1 = 0$
b. about 0.19

5. a. 3
b. $\frac{1}{12}$ or 0.08

6. $\vec{r} = (3, 1, 1) + t(2, -1, 2), t \in \mathbf{R}$

7. a. no solution
b. no solution
c. no solution

8. **a.** $x = -\frac{5}{7}t, y = 1 + \frac{2}{7}t, z = t, t \in \mathbf{R}$
b. $x = 3, y = \frac{1}{4}, z = -\frac{1}{2}$
c. $x = 3t - 3s + 7, y = t, z = s, s, t \in \mathbf{R}$

9. **a.** $x = \frac{1}{2} + \frac{1}{36}t, y = -\frac{1}{2} + \frac{5}{12}t, z = t, t \in \mathbf{R}$
b. $x = \frac{9}{8} - \frac{31}{24}t, y = \frac{1}{4} + \frac{1}{12}t, z = t, t \in \mathbf{R}$

10. **a.** These three planes meet at the point $(-1, 5, 3)$.
b. The planes do not intersect. Geometrically, the planes form a triangular prism.
c. The planes meet in a line through the origin, with equation $x = t, y = -7t, z = -5t, t \in \mathbf{R}$

11. 4.90

12. **a.** $x - 2y + z + 4 = 0$
$\vec{r} = (3, 1, -5) + s(2, 1, 0), s \in \mathbf{R}$
$\vec{m} \cdot \vec{n} = (2, 1, 0)(1, -2, 1) = 0$
Since the line's direction vector is perpendicular to the normal of the plane and the point $(3, 1, -5)$ lies on both the line and the plane, the line is in the plane.
b. $(-1, -1, -5)$
c. $x - 2y + z + 4 = 0$
$-1 - 2(-1) + (-5) + 4 = 0$
The point $(-1, -1, -5)$ is on the plane since it satisfies the equation of the plane.
d. $7x - 2y - 11z - 50 = 0$

13. **a.** 5.48
b. $(3, 0, -1)$

14. **a.** $(-2, -3, 0)$.
b. $\vec{r} = (-2, -3, 0) + t(1, -2, 1), t \in \mathbf{R}$

15. **a.** $-10x + 9y + 8z + 16 = 0$
b. about 0.45

16. **a.** 1
b. $\vec{r} = (0, 0, -1) + t(4, 3, 7), t \in \mathbf{R}$

17. **a.** $x = 2, y = -1, z = 1$
b. $x = 7 - 3t, y = 3 - t, z = t, t \in \mathbf{R}$

18. $a = \frac{2}{3}, b = \frac{3}{4}, c = \frac{1}{2}$

19. $\left(4, -\frac{7}{4}, \frac{7}{2}\right)$

20. $\left(-\frac{5}{3}, \frac{8}{3}, \frac{4}{3}\right)$

21. **a.** $\vec{r} = \left(\frac{45}{4}, 0, -\frac{21}{4}\right) + t(11, 2, -5), t \in \mathbf{R};$
$\vec{r} = \left(-\frac{37}{2}, 0, \frac{15}{2}\right) + t(11, 2, -5), t \in \mathbf{R};$
$\vec{r} = (7, 0, -1) + t(11, 2, -5), t \in \mathbf{R}; z = -1 - 5t, t \in \mathbf{R}$
b. All three lines of intersection found in part a. have direction vector $(11, 2, -5)$, and so they are all parallel. Since no pair of normal vectors for these three planes is parallel, no pair of these planes is coincident.

22. $\left(\frac{1}{2}, 1, \frac{1}{3}\right), \left(\frac{1}{2}, 1, -\frac{1}{3}\right), \left(\frac{1}{2}, -1, \frac{1}{3}\right), \left(\frac{1}{2}, -1, -\frac{1}{3}\right), \left(-\frac{1}{2}, 1, \frac{1}{3}\right), \left(\frac{1}{2}, -1, -\frac{1}{3}\right) \left(-\frac{1}{2}, 1, -\frac{1}{3}\right)$, and $\left(-\frac{1}{2}, -1, \frac{1}{3}\right)$

23. $y = \frac{7}{6}x^2 - \frac{3}{2}x - \frac{2}{3}$

24. $\left(\frac{29}{7}, \frac{4}{7}, -\frac{33}{7}\right)$

25. $A = 5, B = 2, C = -4$

26. **a.** $\vec{r} = (-1, -4, -6) + t(-5, -4, -3), t \in \mathbf{R}$
b. $\left(\frac{13}{2}, 2, -\frac{3}{2}\right)$
c. about 33.26 units^2

27. $6x - 8y + 9z - 115 = 0$

Chapter 9 Test, p. 556

1. **a.** $(3, -1, -5)$
b. $3 - (-1) + (-5) + 1 = 0$
$3 + 1 - 5 + 1 = 0$
$0 = 0$

2. **a.** $\frac{13}{12}$ or 1.08
b. $\frac{40}{3}$ or 13.33

3. **a.** $x = \frac{4t}{5}, y = 1 - \frac{t}{5}, z = t, t \in \mathbf{R}$
b. $(4, 0, 5)$

4. **a.** $(1, -5, 4)$
b. The three planes intersect at the point $(1, -5, 4)$.

5. **a.** $x = -\frac{1}{2} - \frac{t}{4}, y = \frac{3t}{4} + \frac{1}{2}, z = t, t \in \mathbf{R}$
b. The three planes intersect at this line.

6. **a.** $m = -1, n = -3$
b. $x = -1, y = 1 - t, z = t, t \in \mathbf{R}$

7. 10.20

Cumulative Review of Vectors, pp. 557–560

1. **a.** about 111.0°
b. scalar projection: $-\frac{14}{13}$, vector projection: $\left(-\frac{52}{169}, \frac{56}{169}, -\frac{168}{169}\right)$
c. scalar projection: $-\frac{14}{3}$, vector projection: $\left(-\frac{28}{9}, \frac{14}{9}, \frac{28}{9}\right)$

2. **a.** $x = 8 + 4t, y = t, z = -3 - 3t, t \in \mathbf{R}$
b. about 51.9°

3. **a.** $\frac{1}{2}$
b. 3
c. $\frac{3}{2}$

4. **a.** $-7\vec{i} - 19\vec{j} - 14\vec{k}$
b. 18

5. x-axis: about 42.0°, y-axis: about 111.8°, z-axis: about 123.9°

6. **a.** $(-7, -5, -1)$
b. $(-42, -30, -6)$
c. about 8.66 square units
d. 0

7. $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

8. **a.** vector equation: Answers may vary. $\vec{r} = (2, -3, 1) + t(-1, 5, 2), t \in \mathbf{R}$; parametric equation: $x = 2 - t, y = -3 + 5t, z = 1 + 2t, t \in \mathbf{R}$
b. If the x-coordinate of a point on the line is 4, then $2 - t = 4$, or $t = -2$. At $t = -2$, the point on the line is $(2, -3, 1) - 2(-1, 5, 2) = (4, -13, -3)$. Hence, $C(4, -13, -3)$ is a point on the line.

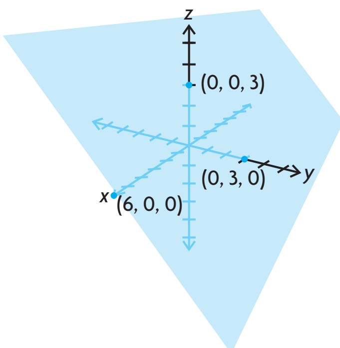
9. The direction vector of the first line is $(-1, 5, 2)$ and of the second line is $(1, -5, -2) = -(-1, 5, 2)$. So they are collinear and hence parallel.
The lines coincide if and only if for any point on the first line and second line, the vector connecting the two points is a multiple of the direction vector for the lines. $(2, 0, 9)$ is a point on the first line and $(3, -5, 10)$ is a point on the second line.
$(2, 0, 9) - (3, -5, 10) = (-1, 5, -1) \neq k(-1, 5, 2)$ for $k \in \mathbf{R}$. Hence, the lines are parallel and distinct.

10. vector equation:
$\vec{r} = (0, 0, 4) + t(0, 1, 1), t \in \mathbf{R}$;
parametric equation: $x = 0, y = t,$
$z = 4 + t, t \in \mathbf{R}$

11. -13

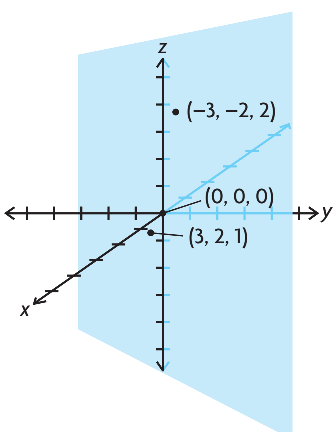
12. $\left(\frac{3}{2}, -\frac{31}{6}, \frac{13}{6}\right)$

13. **a.**



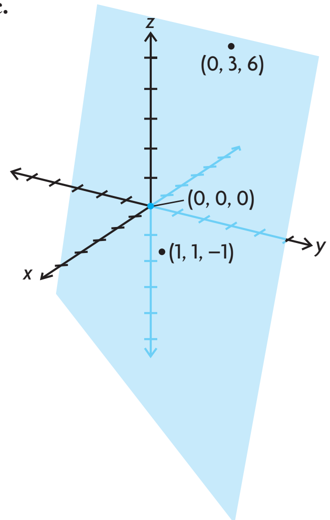
Answers may vary. For example, $(0, 3, -3)$ and $(6, 0, -3)$.

b.



Answers may vary. For example, $(-3, -2, 2)$ and $(3, 2, 1)$.

c.



Answers may vary. For example, $(0, 3, 6)$ and $(1, 1, -1)$.

14. $(-7, 10, 20)$

15. $\vec{q} = (1, 0, 2) + t(-11, 7, 2), t \in \mathbf{R}$

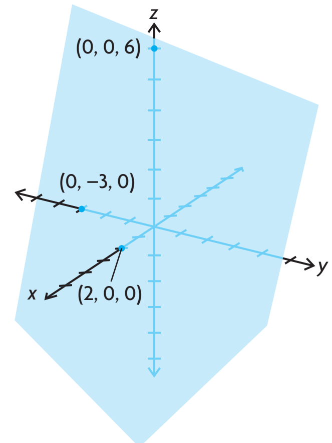
16. **a.** $12x - 9y - 6z + 24 = 0$
b. about 1.49 units

17. **a.** $3x - 5y + 4z - 7 = 0$
b. $x - y + 12z - 27 = 0$
c. $z - 3 = 0$
d. $x + 2z + 1 = 0$

18. 336.80 km/h, N 12.1° W

19. **a.** $\vec{r} = (0, 0, 6) + s(1, 0, -3) + t(0, 1, 2), s, t \in \mathbf{R}$. To verify, find the Cartesian equation corresponding to the above vector equation and see if it is equivalent to the Cartesian equation given in the problem. A normal vector to this plane is the cross product of the two directional vectors.
$\vec{n} = (1, 0, -3) \times (0, 1, 2)$
$= (0(2) - (-3)(1), -3(0) - 1(2), 1(1) - 0(0))$
$= (3, -2, 1)$
So the plane has the form $3x + 2y + z + D = 0$, for some constant D. To find D, we know that $(0, 0, 6)$ is a point on the plane, so $3(0) - 2(0) + (6) + D = 0$. So, $6 + D = 0$, or $D = -6$. So, the Cartesian equation for the plane is $3x - 2y + z - 6 = 0$. Since this is the same as the initial Cartesian equation, the vector equation for the plane is correct.

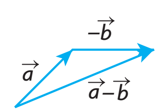
b.



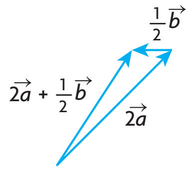
20. **a.** 16°
b. The two planes are perpendicular if and only if their normal vectors are also perpendicular. A normal vector for the first plane is $(2, -3, 1)$ and a normal vector for the second plane is $(4, -3, -17)$. The two vectors are perpendicular if and only if their dot product is zero.
$(2, -3, 1) \cdot (4, -3, -17)$
$= 2(4) - 3(-3) + 1(-17)$
$= 0$
Hence, the normal vectors are perpendicular. Thus, the planes are perpendicular.
c. The two planes are parallel if and only if their normal vectors are also parallel. A normal vector for the first plane is $(2, -3, 2)$ and a normal vector for the second plane is $(2, -3, 2)$. Since both normal vectors are the same, the planes are parallel. Since $2(0) - 3(-1) + 2(0) - 3 = 0$, the point $(0, -1, 0)$ is on the second plane. Yet since $2(0) - 3(-1) + 2(0) - 1 = 2 \neq 0$, $(0, -1, 0)$ is not on the first plane. Thus, the two planes are parallel but not coincident.

21. resultant: about 56.79 N, 37.6° from the 25 N force toward the 40 N force, equilibrant: about 56.79 N, 142.4° from the 25 N force away from the 40 N force

22. **a.**



b.



23. a. $\left(\frac{6}{7}, \frac{2}{7}, -\frac{3}{7}\right)$

b. $\left(-\frac{6}{7}, -\frac{2}{7}, \frac{3}{7}\right)$

24. a. $\overrightarrow{OC} = (8, 9)$,
$\overrightarrow{BD} = (10, -5)$

b. about 74.9°

c. about 85.6°

25. a. $x = t, y = -1 + t, z = 1, t \in \mathbf{R}$

b. $(1, 2, -3)$

c. $x = 1, y = t, z = -3 + t, t \in \mathbf{R}$

d. $x = 1 + 3s + t, y = t, z = s,$
$s, t \in \mathbf{R}$

26. a. yes; $x = 0, y = -1 + t, z = t, t \in \mathbf{R}$

b. no

c. yes;
$x = 2 - 2t, y = t, z = 3t, t \in \mathbf{R}$

27. 30°

28. a. $-\frac{3}{2}$

b. 84

29. $\vec{r} = t(-1, 3, 1), t \in \mathbf{R}$,
$-x + 3y + z - 11 = 0$

30. $(-1, 1, 0)$

31. a. 0.8 km

b. 12 min

32. a. Answers may vary.
$\vec{r} = (6, 3, 4) + t(4, 4, 1), t \in \mathbf{R}$

b. The line found in part a will lie in the plane $x - 2y + 4z - 16 = 0$ if and only if both points $A(2, -1, 3)$ and $B(6, 3, 4)$ lie in this plane. We verify this by substituting these points into the equation of the plane, and checking for consistency.
For A:
$2 - 2(-1) + 4(3) - 16 = 0$
For B:
$6 - 2(3) + 4(4) - 16 = 0$
Since both points lie on the plane, so does the line found in part a.

33. 20 km/h at N 53.1° E

34. parallel: 1960 N,
perpendicular: about 3394.82 N

35. a. True; all non-parallel pairs of lines intersect in exactly one point in $\mathbf{R}^2$. However, this is not the case for lines in $\mathbf{R}^3$ (skew lines provide a counterexample).

b. True; all non-parallel pairs of planes intersect in a line in $\mathbf{R}^3$.

c. True; the line $x = y = z$ has direction vector $(1, 1, 1)$, which is not perpendicular to the normal vector $(1, -2, 2)$ to the plane $x - 2y + 2z = k$, k is any constant. Since these vectors are not perpendicular, the line is not parallel to the plane, and so they will intersect in exactly one point.

d. False; a direction vector for the line $\frac{x}{2} = y - 1 = \frac{z+1}{2}$ is $(2, 1, 2)$. A direction vector for the line $\frac{x-1}{-4} = \frac{y-1}{-2} = \frac{z+1}{-2}$ is $(-4, -2, -2)$, or $(2, 1, 1)$ (which is parallel to $(-4, -2, -2)$). Since $(2, 1, 2)$ and $(2, 1, 1)$ are obviously not parallel, these two lines are not parallel.

36. a. A direction vector for
L_1: $x = 2, \frac{y-2}{3} = z$ is $(0, 3, 1)$,
and a direction vector for
L_2: $x = y + k = \frac{z + 14}{k}$ is $(1, 1, k)$.
But $(0, 3, 1)$ is not a nonzero scalar multiple of $(1, 1, k)$ for any k, since the first component of $(0, 3, 1)$ is 0. This means that the direction vectors for L_1 and L_2 are never parallel, which means that these lines are never parallel for any k.

b. 6; $(2, -4, -2)$

Calculus Appendix

Implicit Differentiation, p. 564

1. The chain rule states that if y is a composite function, then $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$. To differentiate an equation implicitly, first differentiate both sides of the equation with respect to x, using the chain rule for terms involving y, then solve for $\frac{dy}{dx}$.

2. a. $-\frac{x}{y}$

b. $\frac{x^2}{5y}$

c. $\frac{-y^2}{2xy + y^2}$

d. $\frac{9x}{16y}$

e. $-\frac{13x}{48y}$

f. $-\frac{2x}{2y + 5}$

3. a. $y = \frac{2}{3}x - \frac{13}{3}$

b. $y = \frac{2}{3}(x + 8) + 3$

c. $y = -\frac{3\sqrt{3}}{5}x - 3$

d. $y = \frac{11}{10}(x + 11) - 4$

4. $(0, 1)$

5. a. 1

b. $\left(\frac{3}{\sqrt{5}}, \sqrt{5}\right)$ and $\left(-\frac{3}{\sqrt{5}}, -\sqrt{5}\right)$

6. -10

7. $7x - y - 11 = 0$

8. $y = \frac{1}{2}x - \frac{3}{2}$

9. a. $\frac{4}{(x + y)^2} - 1$

b. $4\sqrt{x + y} - 1$

10. a. $\frac{3x^2 - 8xy}{4x^2 - 3}$

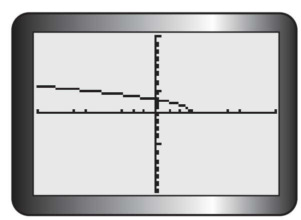
b. $y = \frac{x^3}{4x^2 - 3}; \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$

c. $\frac{dy}{dx} = \frac{3x^2 - 8xy}{4x^2 - 3}$

$y = \frac{x^3}{4x^2 - 3}$

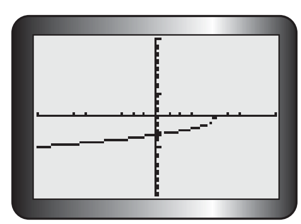
$$\frac{dy}{dx} = \frac{3x^2 - 8x\left(\frac{x^3}{4x^2 - 3}\right)}{4x^2 - 3}$$
$$= \frac{3x^2 - (4x^2 - 3) - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{12x^4 - 9x^2 - 8x^4}{(4x^2 - 3)^2}$$
$$= \frac{4x^4 - 9x^2}{(4x^2 - 3)^2}$$

11. a.



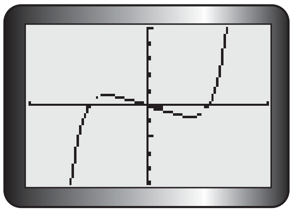
one tangent

b.



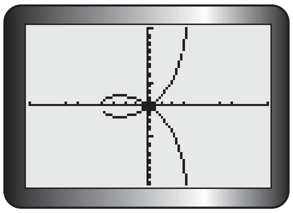
one tangent

c.



one tangent

d.



two tangents

12. $\frac{1}{2}\left(\frac{x}{y}\right)^{-\frac{1}{2}} \frac{1y - \frac{dy}{dx}x}{y^2}$

$+ \frac{1}{2}\left(\frac{y}{x}\right)^{-\frac{1}{2}} \frac{\frac{dy}{dx}x - y}{x^2} = 0$

$\frac{y^{\frac{1}{2}}}{2x^{\frac{1}{2}}} \frac{1y - \frac{dy}{dx}x}{y^2} + \frac{x^{\frac{1}{2}}}{2y^{\frac{1}{2}}} \frac{\frac{dy}{dx}x - y}{x^2} = 0$

Multiply by $2x^2y^2$:

$x^{\frac{3}{2}}y^{\frac{1}{2}}\left(y - x\frac{dy}{dx}\right) + x^{\frac{1}{2}}y^{\frac{3}{2}}\left(\frac{dy}{dx}x - y\right) = 0$

$x^{\frac{3}{2}}y^{\frac{3}{2}} - x^{\frac{5}{2}}y^{\frac{1}{2}}\frac{dy}{dx} + x^{\frac{3}{2}}y^{\frac{3}{2}}\frac{dy}{dx} - x^{\frac{1}{2}}y^{\frac{5}{2}} = 0$

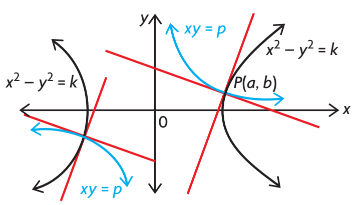
$\frac{dy}{dx}\left(x^{\frac{3}{2}}y^{\frac{3}{2}} - x^{\frac{5}{2}}y^{\frac{1}{2}}\right) = x^{\frac{1}{2}}y^{\frac{5}{2}} - x^{\frac{3}{2}}y^{\frac{3}{2}}$

$\frac{dy}{dx} = \frac{x^{\frac{1}{2}}y^{\frac{3}{2}}(y - x)}{x^{\frac{3}{2}}y^{\frac{1}{2}}(y - x)}$

$\frac{dy}{dx} = \frac{y}{x}$, as required.

13. $2x - 3y + 10 = 0$ and $x = 4$

14.



Let $P(a, b)$ be the point of intersection where $a \neq 0$ and $b \neq 0$.

For $x^2 - y^2 = k$,

$2x - 2y\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{x}{y}$

At $P(a, b)$,

$\frac{dy}{dx} = \frac{a}{b}$

For $xy = P$,

$1 \cdot y + \frac{dy}{dx}x = P$

$\frac{dy}{dx} = -\frac{y}{x}$

At $P(a, b)$,

$\frac{dy}{dx} = -\frac{b}{a}$

At point $P(a, b)$, the slope of the tangent line of $xy = P$ is the negative reciprocal of the slope of the tangent line of $x^2 - y^2 = k$. Therefore, the tangent lines intersect at right angles, and thus, the two curves intersect orthogonally for all values of the constants k and P.

15. $\frac{1}{2}x^{\frac{1}{2}} + \frac{1}{2}y^{\frac{1}{2}}\frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{\sqrt{y}}{\sqrt{x}}$

Let $P(a, b)$ be the point of tangency.

$\frac{dy}{dx} - \frac{\sqrt{b}}{\sqrt{a}}$

Equation on tangent line l and P is

$\frac{y - b}{x - a} = -\frac{\sqrt{b}}{\sqrt{a}}.$

x-intercept is found when $y = 0$.

$\frac{-b}{x - a} = -\frac{\sqrt{b}}{\sqrt{a}}$

$-b\sqrt{a} = -\sqrt{b}x + a\sqrt{b}$

$x = \frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{b}}$

Therefore, the x-intercept is $\frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{b}}$.

For the y-intercept, let $x = 0$,

$\frac{y - b}{-a} = -\frac{\sqrt{b}}{\sqrt{a}}.$

y-intercept is $\frac{a\sqrt{b}}{\sqrt{a}} + b$.

The sum of the intercepts is

$\frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{b}} + \frac{a\sqrt{b} + b\sqrt{a}}{\sqrt{a}}$

$= \frac{a^{\frac{3}{2}}b^{\frac{1}{2}} + 2ab + b^{\frac{3}{2}}a^{\frac{1}{2}}}{a^{\frac{1}{2}}b^{\frac{1}{2}}}$

$= \frac{a^{\frac{1}{2}}b^{\frac{1}{2}}(a + 2\sqrt{a}\sqrt{b} + b)}{a^{\frac{1}{2}}b^{\frac{1}{2}}}$

$= a + 2\sqrt{a}\sqrt{b} + b$

$= (a^{\frac{1}{2}} + b^{\frac{1}{2}})^2$

Since $P(a, b)$ is on the curve, then $\sqrt{a} + \sqrt{b} = \sqrt{k}$, or $a^{\frac{1}{2}} + b^{\frac{1}{2}} = k^{\frac{1}{2}}$.

Therefore, the sum of the intercepts is $(k^{\frac{1}{2}})^2 = k$, as required.

16. $(x + 2)^2 + (y - 5)^5 = 18$ and $(x - 4)^2 + (y + 1)^2 = 18$

Related Rates, pp. 569–570

1. **a.** $\frac{dA}{dt} = 4\ \text{m/s}^2$

b. $\frac{dS}{dt} = -3\ \text{m}^2/\text{min}$

c. $\frac{ds}{dt} = 70$ km/h, when $t = 0.25$

d. $\frac{dx}{dt} = \frac{dy}{dt}$

e. $\frac{d\theta}{dt} = \frac{\pi}{10}$ rad/s

2. **a.** decreasing at 5.9 °C/s

b. about 0.58 m

c. Solve $T''(x) = 0$.

3. area increasing at 100 cm^2/s; perimeter increasing at 20 cm/s

4. **a.** increasing at 300 cm^3/s

b. increasing at 336 cm^2/s

5. increasing at 40 cm^2/s

6. **a.** $\frac{5}{6\pi}$ km/h

b. $\frac{5}{3\pi}$ m/s

7. $\frac{1}{\pi}$ km/h

8. 4 m/s

9. 8 m/min

10. 214 m/s

11. $5\sqrt{13}$ km/h

12. **a.** $\frac{1}{72\pi}$ cm/s

b. $\frac{2}{49\pi}$ cm/s or about 0.01 cm/s

c. $\frac{1}{8\pi}$ cm/s or about 0.04 cm/s

13. $\frac{50}{\pi}$ cm/min; 94.25 min (or about 1.5 h)

14. Answers may vary. For example:

a. The diameter of a right-circular cone is expanding at a rate of 4 cm/min. Its height remains constant at 10 cm. Find its radius when the volume is increasing at a rate of $80\pi\ \text{cm}^3$/min.

b. Water is being poured into a right-circular tank at the rate of $12\pi\ \text{m}^3$/min. Its height is 4 m and its radius is 1 m. At what rate is the water level rising?

c. The volume of a right-circular cone is expanding because its radius is increasing at 12 cm/min and its height is increasing at 6 cm/min. Find the rate at which its volume is changing when its radius is 20 cm and its height is 40 cm.

15. $0.145\pi\ \text{m}^3$/year

16. $\frac{2}{\pi}$ cm/min

17. $\frac{\sqrt{3}}{4}$ m/min

18. 144 m/min

19. 62.8 km/h

20. a. $\frac{4}{5\pi}$ cm/s

b. $\frac{8}{25\pi}$ cm/s

21. a. $x^2 + y^2 = \left(\frac{l}{2}\right)^2$

b. $\frac{y^2}{k^2} + \frac{y^2}{(l-k)^2} = 1$

The Natural Logarithm and its Derivative, p. 575

1. A natural logarithm has base e; a common logarithm has base 10.

2. Since $e = \lim_{h\to 0}(1 + h)^{\frac{1}{h}}$, let $h = \frac{1}{n}$.
Therefore,
$$e = \lim_{\frac{1}{n}\to 0}\left(1 + \frac{1}{n}\right)^n.$$
But as $\frac{1}{n} \to 0$, $n \to \infty$.
Therefore, $e = \lim_{n\to\infty}(1 + \frac{1}{n})^n$.
If $n = 100$, $e \doteq \left(1 + \frac{1}{100}\right)^{100}$
$= 1.01^{100}$
$\doteq 2.704\ 81$
Try $n = 100\ 000$, etc.

3. a. $\frac{5}{5x + 8}$

b. $\frac{2x}{x^2 + 1}$

c. $\frac{15}{t}$

d. $\frac{1}{2(x + 1)}$

e. $\frac{3t^2 - 4t}{t^3 - 2t^2 + 5}$

f. $\frac{2z + 3}{2(z^2 + 3z)}$

4. a. $\ln x + 1$

b. 1

c. $e^t \ln t + \frac{e^t}{t}$

d. $\frac{-ze^{-z}}{e^{-z} + ze^{-z}}$

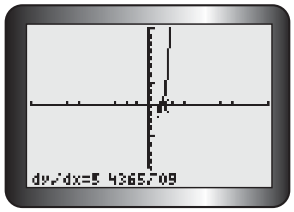
e. $\frac{te^t \ln t - e^t}{t(\ln t)^2}$

f. $\frac{1}{2}e^{\sqrt{u}}\left(\frac{1}{2}e^{\sqrt{u}}\ln u + \frac{1}{u}\right)$

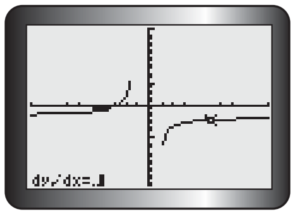
5. a. $2e$

b. 0.1

c.



The value shown is approximately $2e$, which matches the calculation in part a.



This value matches the calculation in part b.

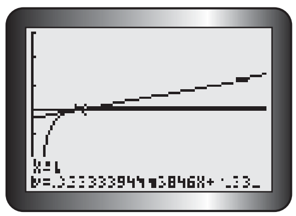
6. a. $x = 0$

b. no solution

c. $x = 0, \pm\sqrt{e - 1}$

7. a. $x - 3y - 1 = 0$

b.

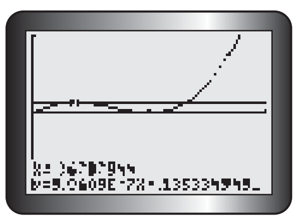


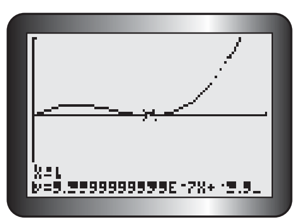
c. The equation on the calculator is in a different form, but is equivalent to the equation in part a.

8. $x - 2y + (2\ln 2 - 4) = 0$

9. a. $\left(\frac{1}{e}, \frac{1}{e^2}\right)$ and $(1, 0)$

b.





c. The solution in part a is more precise and efficient.

10. $y = -\frac{1}{2}x + \ln 2$

11. a. 90 km/h

b. $\frac{-90}{3t + 1}$

c. about -12.8 km/h/s

d. 6.36 s

12. $\frac{1}{2}$

13. a. $\frac{1}{x\ln x}$

b. The function's domain is $\{x\in\mathbf{R} \mid x > 1\}$.
The domain of the derivative is $\{x\in\mathbf{R} \mid x > 0 \text{ and } x \neq 1\}$.

The Derivatives of General Logarithmic Functions, p. 578

1. a. $\frac{1}{x\ln 5}$

b. $\frac{1}{x\ln 3}$

c. $\frac{2}{x\ln 4}$

d. $\frac{-3}{x\ln 7}$

e. $\frac{-1}{x\ln 10}$

f. $\frac{3}{x\ln 6}$

2. a. $\frac{1}{(x + 2)\ln 3}$

b. $\frac{1}{x\ln 8}$

c. $\frac{-6}{(2x + 3)\ln 3}$

d. $\frac{-2}{(5 - 2x)\ln 10}$

e. $\frac{2}{(2x + 6)\ln 8} = \frac{1}{(x + 3)\ln 8}$

f. $\frac{2x + 1}{(x^2 + x + 1)\ln 7}$

3. a. $\frac{5}{52\ln 2}$

b. $\frac{1}{8\log_2(8)(\ln 3)(\ln 2)}$

4. a. $\frac{2}{(1 - x^2)\ln 10}$

b. $\frac{2x + 3}{2(x^2 + 3x)\ln(2)}$

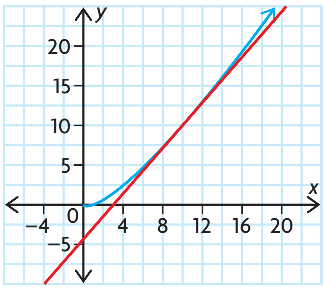
c. $\frac{2\ln 5 - \ln 4}{\ln 3}$

d. $\frac{x\ln 3(3^x)(\ln x) + 3^x}{x\ln 3}$

e. $\dfrac{\ln x + 1}{\ln 2}$

f. $\dfrac{4x + 1 - x\ln(3x^2)}{2x \ln 5(x + 1)^{\frac{3}{2}}}$

5. $y = 1.434x - 4.343$



6. $y = \log_a kx$

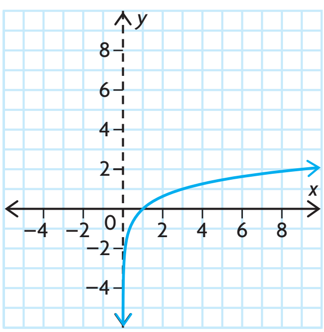
$$\frac{dy}{dx} = \frac{f'(x)}{f(x)\ln(a)} = \frac{k}{kx\ln(a)} = \frac{1}{x\ln(a)}$$

7. $y = 49.1x - 235.5$

8. Since the derivative is positive at $t = 15$, the distance is increasing at that point.

9. **a.** $y = 0.1x + 1.1$

b.



vertical asymptote at $x = 0$

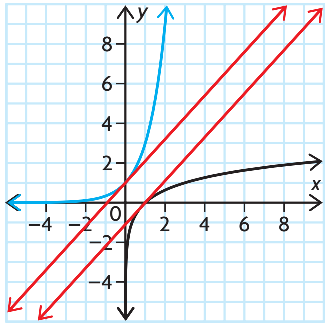
c. The tangent line will intersect this asymptote because it is defined for $x = 0$.

10. $D = \{x \in \mathbf{R} | x < -2 \text{ or } x > 2\}$; critical number at $x = 0$, $x = 2$, and $x = -2$; function is decreasing for $x < -2$ and increasing for $x > 2$

11. **a.** point of inflection at $x = 0$

b. $x = 0$ is a possible point of inflection. Since the graph is always concave up, there is no point of inflection.

12. The slope of $y = \log_3 x$ at $(1, 0)$ is $\frac{1}{\ln 3}$. Since $\ln 3 > 1$, the slope of $y = 3^x$ at $(0, 1)$ is greater than the slope of $y = \log_3 x$ at $(1, 0)$.



Logarithmic Differentiation, p. 582

1. **a.** $\sqrt{10}x^{\sqrt{10}-1}$

b. $15\sqrt{2}\,x^{3\sqrt{2}-1}$

c. $\pi t^{\pi - 1}$

d. $ex^{e-1} + e^x$

2. **a.** $\dfrac{2x^{\ln x}\ln x}{x}$

b. $\dfrac{(x+1)(x-3)^3}{(x+2)^3} \times \left(\dfrac{1}{x+1} + \dfrac{2}{x-3} - \dfrac{3}{x+2}\right)$

c. $\left(x^{\sqrt{x}}\right)\dfrac{\ln x + 2}{2\sqrt{x}}$

d. $\left(\dfrac{1}{t}\right)^t\left(\ln\dfrac{1}{t} - 1\right)$

3. **a.** $2e^e$

b. $e^2 + e \cdot 2^{e-1}$

c. $-\dfrac{4}{27}$

4. $y = 32(2\ln 2 + 1)(x - 128\ln 2 - 48)$

5. $-\dfrac{11}{36}$

6. $(e, e^{\frac{1}{e}})$

7. $(1, 1)$ and $(2, 4 + 4\ln 2)$

8. $\dfrac{32(\ln 4 + 1)^2}{\ln 4 + 2}$

9. $\dfrac{1}{8}$

10. $\left(\dfrac{x\sin x}{x^2 - 1}\right)^2 \times \left(\dfrac{2(\sin x + x\cos x)}{x\sin x} - \dfrac{4x}{x^2 - 1}\right)$

11. $x^{\cos x}\left(\sin x \ln x + \dfrac{\cos x}{x}\right)$

12. $y = x$

13. **a.** $v(t) = t^{\frac{1}{t}}\left(\dfrac{1 - \ln t}{t^2}\right)$,

$a(t) = \dfrac{t^{\frac{1}{t}}}{t^4}[1 - 2\ln t + (\ln t)^2 + 2t\ln t - 3t]$

b. $t = e$; $a(e) = -e^{\frac{1}{e} - 3}$

14. Using a calculator, $e^\pi \doteq 23.14$ and $\pi^e \doteq 22.46$. So, $e^\pi > \pi^e$.

Vector Appendix

Gaussian Elimination, pp. 588–590

1. **a.** $\left[\begin{array}{rrr|r} 1 & 2 & -1 & -1 \\ -1 & 3 & -2 & -1 \\ 0 & 3 & -2 & -3 \end{array}\right]$

b. $\left[\begin{array}{rrr|r} 2 & 0 & -1 & 1 \\ 0 & 2 & -1 & 16 \\ -3 & 1 & 0 & 10 \end{array}\right]$

c. $\left[\begin{array}{rrr|r} 2 & -1 & -1 & -2 \\ 1 & -1 & 4 & -1 \\ -1 & -1 & 0 & 13 \end{array}\right]$

2. Answers may vary. For example:

$\left[\begin{array}{rr|r} 1 & 1.5 & 0 \\ 0 & -5.5 & 1 \end{array}\right]$

$\left[\begin{array}{rr|r} 2 & 3 & 0 \\ 0 & -5.5 & 1 \end{array}\right]$

3. Answers may vary. For example:

$\left[\begin{array}{rrr|r} 2 & 1 & 6 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & -37 & 4 \end{array}\right]$

4. **a.** Answers may vary. For example:

$\left[\begin{array}{rrr|r} 1 & 0 & -1 & -1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & -36 & 16 \end{array}\right]$

b. $x = -\dfrac{22}{9}, y = -\dfrac{8}{9}, z = -\dfrac{4}{9}$

5. **a.** $x - 2y = -1$
$2x - 3y = 1$
$2x - y = 0$

b. $-2x - z = 0$
$x - 2y = 4$
$y + 2z = -3$

c. $-z = 0$
$x = -2$
$y + z = 0$

6. **a.** $x = -\dfrac{9}{2}, y = -3$

b. $x = 13, y = 9, z = -6$

c. no solution

d. $x = -\dfrac{9}{4}, y = -4, z = -5$

e. $x = 2 - 3t + s, y = s,$
$z = t, s, t \in \mathbf{R}$
f. $x = 4, y = 8, z = -2$

7. **a.** It satisfies both properties of a matrix in row-echelon form.
1. All rows that consist entirely of zeros must be written at the bottom of the matrix.
2. In any two successive rows not consisting entirely of zeros, the first nonzero number in the lower row must occur further to the right that the first nonzero number in the row directly above.

b. A solution does not exist to this system, because the second row has no variables, but is still equal to a nonzero number, which is not possible.
c. Answers may vary. For example:
$$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 3 \\ -2 & 2 & 2 & 3 \\ -1 & 1 & 1 & 3 \end{array}\right]$$

8. **a.** no; Answers may vary. For example:
$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 1 \end{array}\right]$$
b. no; Answers may vary. For example:
$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -3 \\ 0 & 1 & -10 & 11 \\ 0 & 0 & 3 & 6 \end{array}\right]$$
c. no; Answers may vary. For example:
$$\left[\begin{array}{ccc|c} -1 & 2 & 1 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 \end{array}\right]$$
d. yes

9. **a.** **i.** $x = -\frac{5}{2}, y = 0, z = \frac{1}{2}$
ii. $x = -7, y = 31, z = 2$
iii. $x = 2t - 6, y = t, z = -6,$
$t \in \mathbf{R}$
iv. $x = -12 - 9t, y = -3 - 2t,$
$z = t, t \in \mathbf{R}$
b. **i.** The solution is the point at which the three planes meet.
ii. The solution is the point at which the three planes meet.
iii. The solution is the line at which the three planes meet.
iv. The solution is the line at which the three planes meet.

10. **a.** $x = -3, y = -4, z = 10$
The three planes meet at the point $(-3, -4, 10)$.
b. $x = -2t, y = t, z = t, t \in \mathbf{R}$
The three planes meet at this line.
c. $x = -1, y = 3t, z = t, t \in \mathbf{R}$
The three planes meet at this line.
d. $x = 0, y = 4, z = -2$
The three planes meet at the point $(0, 4, -2)$.
e. $x = -\frac{1}{2}, y = 2 - t, z = t, t \in \mathbf{R}$
The three planes meet at this line.
f. $x = 500, y = 1000, z = -1500$
The three planes meet at the point $(500, 1000, -1500)$.

11. $x = \frac{7a - 3c + 5b}{3},$
$y = \frac{3c - 4b - 5a}{3}, z = c - b - 2a$

12. $y = 2x^2 + 7x - 2$

13. $p = \frac{143}{9}, q = \frac{9}{121}, r = 33$

14. **a.** $a = -2$
b. $a = 1$
c. $a \neq -2$ or $a \neq 1$

Gauss-Jordan Method for Solving Systems of Equations, pp. 594–595

1. **a.** $\left[\begin{array}{cc|c} 1 & 0 & -7 \\ 0 & 1 & -2 \end{array}\right]$

b. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{array}\right]$

c. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$

d. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -4 \end{array}\right]$

2. **a.** $(-7, -2)$
b. $(3, 2, 0)$
c. $(1, 1, 0)$
d. $(8, -1, -4)$

3. **a.** $x = -1, y = 10, z = 11$
b. $x = 3, y = 5, z = 7$
c. $x = 1, y = 2, z = -4$
d. $x = 0, y = 0, z = -1$
e. $x = -4, y = \frac{1}{3}, z = 0$
f. $x = \frac{1}{2}, y = \frac{1}{3}, z = \frac{1}{6}$

4. **a.** $x = -1, y = 2, z = 6$
b. $x = 38, y = 82, z = 14$

5. **a.** $k = 3$
b. $k \neq 3, k \in \mathbf{R}$
c. The matrix cannot be put in reduced row-echelon form.

6. **a.** Every homogeneous system has at least one solution, because (0, 0, 0) satisfies each equation.
b. $\left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & 0 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$
The reduced row-echelon form shows that the intersection of these planes is a line that goes through the point $(0, 0, 0)$.
$x = -\frac{2}{3}t, y = -\frac{1}{3}t, z = t, t \in \mathbf{R}$

7. $(2, 3, 6)$