Answers

Chapter 1

Review of Prerequisite Skills, pp. 2–3

1. a. -3  c. 4  e. -4.1
   b. -2  d. -4  f. $\frac{-1}{2}$

2. a. $y = 4x - 2$
   b. $y = -2x + 5$
   c. $y = \frac{6}{5}(x + 1) + 6$
   d. $x + y = -2 = 0$
   e. $x = -3$
   f. $y = 5$

3. a. $-1$  c. -9
   b. 0  d. 144

4. a. $\frac{5}{2}$  c. 0
   b. $\frac{3}{4}$  d. $\frac{5}{2}$

5. a. 6  c. 9
   b. $\sqrt{3}$  d. $\sqrt{6}$

6. a. $\frac{-1}{2}$  c. 5  e. 10^6
   b. 1  d. 1

7. a. $x^2 - 4x - 12$
   b. $15 + 17x - 4x^2$
   c. $-x^2 - 7x$
   d. $-x^2 + x + 7$
   e. $a^3 + 6a^2 + 12a + 8$
   f. $729a^3 - 1215a^2 + 675a - 125$

8. a. $(x + 1)(x - 1)$
   b. $(x + 3)(x - 2)$
   c. $(2x - 3)(x - 2)$
   d. $(x + 1)(x + 1)$
   e. $(3x - 4)(9x^2 + 12x + 16)$
   f. $(x - 1)(2x - 3)(x + 2)$

9. a. $x \in \mathbb{R} \mid x \neq -5$
   b. $x \in \mathbb{R}$
   c. $x \in \mathbb{R} \mid x \neq 1$
   d. $x \in \mathbb{R} \mid x \neq 0$
   e. $\left\{ x \in \mathbb{R} \mid x \neq -\frac{1}{2}, 3 \right\}$
   f. $\left\{ x \in \mathbb{R} \mid x \neq -5, -2, 1 \right\}$

10. a. 20.1 m/s  b. 10.3 m/s

11. a. $-20$ L/min
   b. about $-13.33$ L/min
   c. The volume of water in the hot tub is always decreasing during that time period, a negative change.

Section 1.1, p. 9

12. a. $m = -8$
   c. $-8$

Section 1.2, pp. 18–21

1. a. $-\frac{5}{3}$  b. $\frac{7}{13}$

2. a. $\frac{1}{\sqrt{a + 2}}$
   b. $\frac{1}{\sqrt{x + 4} + 2}$
   c. $\frac{1}{\sqrt{x + h + \sqrt{x}}}$

3. a. $7x - 17y - 40 = 0$

4. a. $\frac{1}{\sqrt{5} + 1}$
   b. $\frac{2}{2 + 3\sqrt{2}}$
   c. $\frac{1}{12 - 5\sqrt{3}}$

5. a. $8\sqrt{10} + 24$
   b. $8\sqrt{10} + 24$
   c. The expressions are equivalent. The radicals in the denominator of part a. have been simplified in part b.

6. a. $\sqrt{6} + 2$
   c. $2\sqrt{3} + \sqrt{6}$
   d. $\frac{12 + 5\sqrt{6}}{2}$

7. a. $\frac{12\sqrt{15} + 15\sqrt{10}}{2}$
   f. $5 + 2\sqrt{2}$
c. 3x − 5y − 15 = 0

d. x = 5

4. a. 75 + 15h + h²
b. 108 + 54h + 12h² + h³
c. \[ \frac{1}{1 + h} \]
d. 6 + 3h
e. \[ \frac{5}{4(4 + h)} \]
f. \[ \frac{1}{4 + 2h} \]

5. a. \[ \frac{1}{\sqrt{16 + h + 4}} \]
b. \[ \frac{1}{\sqrt{h^2 + 5h + 4}} \]
c. \[ \frac{1}{\sqrt{5 + h + \sqrt{5}}} \]

6. a. \[ \frac{6 + 3h}{h + 1} \]
b. \[ \frac{3 + 3h + h^2}{h + 2} \]
c. \[ \frac{1}{\sqrt{9 + h + 3}} \]

7. a.  
<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>Slope of Line</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2, 8)</td>
<td>(3, 27)</td>
<td>19</td>
</tr>
<tr>
<td>(2, 8)</td>
<td>(2.5, 15.625)</td>
<td>15.25</td>
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<tr>
<td>(2, 8)</td>
<td>(2.1, 9.261)</td>
<td>12.61</td>
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<tr>
<td>(2, 8)</td>
<td>(2.01, 8.120 601)</td>
<td>12.0601</td>
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<td>(2, 8)</td>
<td>(1, 1)</td>
<td>7</td>
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<tr>
<td>(2, 8)</td>
<td>(1.5, 3.375)</td>
<td>9.25</td>
</tr>
<tr>
<td>(2, 8)</td>
<td>(1.9, 6.859)</td>
<td>11.41</td>
</tr>
<tr>
<td>(2, 8)</td>
<td>(1.99, 7.880 599)</td>
<td>11.9401</td>
</tr>
</tbody>
</table>

b. 12
c. 12 + 6h + h²
d. 12
e. They are the same.
f.  They are the same.

8. a. −12
b. 5
c. 12

9. a. \[ \frac{1}{2} \]
b. \[ \frac{1}{4} \]
c. \[ \frac{5}{6} \]

10. a. −2
b. \[ \frac{1}{2} \]
c. \[ −\frac{1}{25} \]

11. a. 1
d. \[ \frac{1}{6} \]
b. −1
e. \[ −\frac{3}{4} \]
c. 9
f. \[ −\frac{1}{6} \]

12.  
\[ y = \sqrt{25 - x^2} \]

13. Semi-circle centre (0, 0), rad 5, y ≥ 0
OA is a radius. The slope of OA is \[ \frac{4}{5} \].
The slope of tangent is \[ \frac{3}{2} \].

14. Since the tangent is horizontal, the slope is 0.

15. 3x − y − 8 = 0
16. 3x + y − 8 = 0
17. a. (3, −2)
b. (5, 6)
c. y = 4x − 14
d. y = 2x − 8
e. y = 6x − 24

18. a. undefined
23. The slope of the tangent at is 0.5.

22. The points of intersection are 0 and 2.
   
21. a. The tangent to is: 
   
20. The coordinates of the point are \( (a, \frac{1}{a}) \).

19. The slope of the tangents at is \(-\frac{1}{a^2}\).

18. The slope of the tangent at is \(-\frac{1}{a^2}\).

17. a. The slope of the tangent is \(-\frac{1}{a^2}\).

16. a. \( \frac{1}{10} \)
   
15. b. \(-1\)
   
14. a. \$4800
   
13. 2 s; 0 m/s

12. \(-\frac{12}{5}\) °C/km

11. \(\frac{1}{50}\) s/m

10. a. 8a + 5
   
9. a. i. 8. Speed is represented only by a number, not a direction.
   
8. Yes, velocity needs to be described by a number and a direction. Only the speed of the school bus was given, not the direction, so it is not correct to use the word “velocity.”

7. a. First second = -5 m/s, third second = -25 m/s, eighth second = -75 m/s
   
6. Yes, velocity needs to be described by a number and a direction. Only the speed of the school bus was given, not the direction, so it is not correct to use the word “velocity.”

5. Speed is represented only by a number, not a direction.

4. a. A and B; greater; the secant line through these two points is steeper than the tangent line at R.
   
3. Slope of the tangent at the point \((6, a(6))\) to the function with equation \(y = \sqrt{x}\) at the point \((4, 2)\). 

2. a. Slope of the secant between the points \((2, a(2))\) and \((9, a(9))\).
   
1. a. 0 s or 4 s

The slope of the tangent at \(a = 0.5\) is 

\(-1 = \frac{m_p}{2} \) and at \(a = -0.5\) is \(1 = m_q\).

Therefore, the tangents are perpendicular at the points of intersection.

24. \(y = -11x + 24\)

25. a. \((0, -2)\)
   
Section 1.3, pp. 29–31

21. a. 75 m
   
20. 500 papers/year

19. \(\frac{5}{4}\)

18. \(\frac{1}{2}\) or \(x = -\frac{1}{2}\)

The points of intersection are \(P\left(\frac{1}{2}, \frac{1}{4}\right)\) and \(Q\left(-\frac{1}{2}, \frac{3}{4}\right)\).

Tangent to \(y = x: \)

\[ m = \lim_{h \to 0} \frac{(a + h)^2 - a^2}{h} = \lim_{h \to 0} \frac{2ah + h^2}{h} = 2a \]

The slope of the tangent at \(a = \frac{1}{2}\) is 

\(1 = m_p\) and at \(a = -\frac{1}{2}\) is \(-1 = m_q\).

Tangents to \(y = \frac{1}{2} - x^2: \)

\[ m = \lim_{h \to 0} \frac{\left[\frac{1}{2} - (a + h)^2\right] - \left[\frac{1}{2} - a^2\right]}{h} = \lim_{h \to 0} \frac{-2ah - h^2}{h} = -2a \]

Mid-Chapter Review, pp. 32–33

2. a. \(\frac{6\sqrt{3} + \sqrt{6}}{3}\)
   
1. a. 3  
   
   b. 37  
   
   c. 5
3. a. \( \frac{2}{5\sqrt{2}} \)
   b. \( \frac{3}{\sqrt{3}(6 + \sqrt{2})} \)
   c. \( -\frac{5}{13} \)
   d. \( -\frac{1}{3\sqrt{2}(2\sqrt{3} + 5)} \)
4. a. \( \frac{2}{3} + y - 6 = 0 \)
   b. \( x - y + 5 = 0 \)
   c. \( 4x - y - 2 = 0 \)
   d. \( x - 5y - 9 = 0 \)
5. \(-2\)
6. a.

<table>
<thead>
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<th>( P )</th>
<th>( Q )</th>
<th>Slope of Line ( PQ )</th>
</tr>
</thead>
<tbody>
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<td>((-2, 6))</td>
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<td>((-1, 1))</td>
<td>((-1.5, 3.25))</td>
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<td>((-1, 1))</td>
<td>((-1.1, 1.41))</td>
<td>(-4.1)</td>
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<td>((-1, 1))</td>
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<tr>
<td>((-1, 1))</td>
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<td>(-4.001)</td>
</tr>
</tbody>
</table>

b. \(-4\)

c. \( h = -4\)

d. \(-4\)

e. The answers are equal.

7. a. \(-3\)
   b. \(-9\)
   c. \(-\frac{1}{4}\)
   d. \(\frac{1}{6}\)

8. a. i. \(36 \text{ km/h}\)
   ii. \(30.6 \text{ km/h}\)
   iii. \(30.06 \text{ km/h}\)
   b. velocity of car appears to approach \(30 \text{ km/h}\)
   c. \((6h + 30) \text{ km/h}\)
   d. \(30 \text{ km/h}\)

9. a. \(-4\)
   b. \(-12\)
10. a. \(-2000 \text{ L/min}\)
    b. \(-1000 \text{ L/min}\)
11. a. \(-9x + y + 19 = 0\)
    b. \(8x + y + 15 = 0\)
    c. \(4x + y + 8 = 0\)
    d. \(-2x + y + 2 = 0\)
    e. \(-3x + 4y - 25 = 0\)
    f. \(3x + 4y + 5 = 0\)

Section 1.4, pp. 37–39

1. a. \(\frac{72}{99}\)
   b. \(\pi\)

2. Evaluate the function for values of the independent variable that get progressively closer to the given value of the independent variable.

3. a. A right-sided limit is the value that a function gets close to as the values of the independent variable decrease and get close to a given value.
   b. A left-sided limit is the value that a function gets close to as the values of the independent variable increase and get close to a given value.
   c. A two-sided limit is the value that a function gets close to as the values of the independent variable get close to a given value, regardless of whether the values increase or decrease toward the given value.

4. a. \(-5\)
   b. \(-10\)
   c. \(-\frac{1}{4}\)
   d. \(-\frac{1}{2}\)
   e. \(4\)
   f. \(8\)

5. \(1\)

6. a. \(0\)
   b. \(2\)
   c. \(-1\)
   d. \(2\)

7. a. \(2\)
   b. \(1\)
   c. does not exist

8. a. \(8\)
   b. \(2\)
   c. \(2\)

9.\[ \text{Graph of a function}\]

10. a. \(0\)
    b. \(0\)
    c. \(\frac{1}{2}\)
    d. \(\frac{1}{5}\)
    e. \(5\)
    f. does not exist; substitution causes division by zero, and there is no way to remove the factor from the denominator.

11. a. does not exist
   b. \(2\)
   c. \(2\)
   d. does not exist

12. Answers may vary. For example:
   a. \[\text{Graph of a function}\]

b. \[\text{Graph of a function}\]

c. \[\text{Graph of a function}\]
Section 1.5, pp. 45–47

1. \[ \lim_{x \to 2} (3 + x) \] and \[ \lim_{x \to 2} (x + 3) \] have the same value, but \[ \lim_{x \to 2} 3 + x \] does not.
Since there are no brackets around the expression, the limit only applies to 3, and there is no value for the last term, \( x \).

2. Factor the numerator and denominator. Cancel any common factors. Substitute the given value of \( x \).

3. Yes, if the two one-sided limits have the same value, then the value of the limit is equal to the value of the one-sided limits.
If the one-sided limits do not have the same value, then the limit does not exist.

4. a. 1  
   b. 1  
   c. \( \frac{100}{9} \)  
   d. 5\( \pi \)  
   e. 2  
   f. \( \sqrt{3} \)

5. a. 2  
   b. \( \sqrt{2} \)

6. Since substituting \( t = 1 \) does not make the denominator 0, direct substitution works.

7. a. 4  
   b. 1  
   c. 27  
   d. \( -\frac{1}{4} \)  
   e. \( \frac{1}{4} \)  
   f. \( \frac{1}{\sqrt{7}} \)

8. a. \( \frac{1}{12} \)  
   b. \( -\frac{27}{4} \)  
   c. \( \frac{1}{6} \)  
   d. \( \frac{1}{2} \)  
   e. \( \frac{1}{12} \)  
   f. \( \frac{1}{32} \)

9. a. 0  
   b. \( \frac{1}{2} \)  
   c. \( 2x \)  
   d. \( \frac{1}{2} \)  
   e. \( \frac{1}{2} \)  
   f. \( \frac{1}{32} \)

10. a. does not exist  
    b. does not exist  
    c. does not exist

11. a. \( \Delta V \) is constant; therefore, \( T \) and \( V \) form a linear relationship.
    b. \( V = 0.082137 + 22.4334 \)  
    c. \( T = \frac{V - 22.4334}{0.082137} \)  
    d. \( \lim_{s \to 0} T = \frac{0 - 22.4334}{0.082137} = -273.145 \)  
    e.  

12. \( \lim_{s \to 5} \frac{x^2 - 4}{f(x)} \)  
    \( = \lim_{s \to 5} \frac{x^2 - 4}{f(x)} \)  
    \( = \frac{21}{3} \)  
    \( = 7 \)
Section 1.6, pp. 51–53

1. Anywhere that you can see breaks or jumps is a place where the function is not continuous.
2. On a given domain, you can trace the graph of the function without lifting your pencil.
3. point discontinuity

4. a. $x = 3$
   b. $x = 0$
   c. $x = 0$
   d. $x = 3$ and $x = -3$
   e. $x = -3$ and $x = 2$
   f. $x = 3$

5. a. continuous for all real numbers
   b. continuous for all real numbers
   c. continuous for all real numbers, except 0 and 5
   d. continuous for all real numbers greater than or equal to $-2$
   e. continuous for all real numbers
   f. continuous for all real numbers

6. $g(x)$ is a linear function (a polynomial), and so is continuous everywhere, including $x = 2$.

7. Yes, the function is continuous everywhere.

8. The function is discontinuous at $x = 0$.

9. Discontinuities at 0, 100, 200, and 500

10. no

11. Discontinuous at $x = 2$

12. $k = 16$

13. a. $g(x)$ is discontinuous at $x = 1$.
Review Exercise, pp. 56–59

1. a. −3  b. 7  c. 2x − y − 5 = 0
2. a. −\frac{1}{3}  b. \frac{1}{2}  c. \frac{1}{27}  d. −\frac{5}{4}
3. a. 2  b. 2
4. a. 1st second = −5 m/s, 2nd second = −15 m/s  b. −40 m/s  c. −60 m/s
5. a. 0.0601 g  b. 6.01 g/min  c. 6 g/min
6. a. 700 000 t  b. 18 \times 10^3 t per year  c. 15 \times 10^3 t per year  d. 7.5 years
7. a. 10  b. 7; 0  c. y = 3 and t = 4
8. a. Answers may vary. For example:

b. Answers may vary. For example:

9. a. \begin{align*}
    x &= -1 \text{ and } x = 1 \\
    \text{b. They do not exist.}
\end{align*}

10. a. x = 1 and x = −2  b. \lim_{x \to -1} f(x) = \frac{2}{3}  \\
    \lim_{x \to -2} f(x) \text{ does not exist.}
11. a. \lim_{x \to 0} f(x) \text{ does not exist.}  \\
    b. \lim_{x \to 0} g(x) = 0  \\
    c. \lim_{x \to -3} h(x) = \frac{37}{7}  \\
    \lim_{x \to -3} h(x) \text{ does not exist.}

12. a. Answers may vary. For example:

13. a. \begin{align*}
    x & \quad 1.9 \quad 1.99 \quad 1.999 \quad 2.001 \quad 2.01 \quad 2.1 \\
    \frac{x - 2}{x^2 - x - 2} & \quad 0.34483 \quad 0.33445 \quad 0.33344 \quad 0.33322 \quad 0.33223 \quad 0.32258 \\
    \frac{1}{3} & \quad \\
\end{align*}

b. \begin{align*}
    x & \quad 0.9 \quad 0.99 \quad 0.999 \quad 1.001 \quad 1.01 \quad 1.1 \\
    \frac{x - 1}{x^2 - 1} & \quad 0.52632 \quad 0.50251 \quad 0.50025 \quad 0.49975 \quad 0.49751 \quad 0.47619 \\
    \frac{1}{3} & \quad \\
\end{align*}

14. \begin{align*}
    x & \quad -0.1 \quad -0.01 \quad -0.001 \quad 0.001 \quad 0.01 \quad 0.1 \\
    \frac{\sqrt{x + 3} - \sqrt[3]{3}}{x} & \quad 0.29112 \quad 0.28892 \quad 0.2887 \quad 0.28865 \quad 0.28843 \quad 0.28631 \\
    \sqrt[3]{\frac{1}{3}} & \quad \\
\end{align*}

15. a. \begin{align*}
    x & \quad 2.1 \quad 2.01 \quad 2.001 \quad 2.0001 \\
    f(x) & \quad 0.24846 \quad 0.24984 \quad 0.24998 \quad 0.25 \\
    \lim_{x \to 2} f(x) & \neq 0.25  \\
\end{align*}

b. \lim_{x \to 2} f(x) = 0.25  \\
c. 0.25

16. a. 10  b. \frac{1}{4}  c. −\frac{1}{16}
17. a. 4  b. \frac{1}{\sqrt{5}}  c. −\frac{1}{8}
   b. \frac{1}{3}  d. \frac{1}{4}

18. a. The function is not defined for x < 3, so there is no left-side limit. 
    b. Even after dividing out common factors from numerator and denominator, there is a factor of x − 2 in the denominator; the graph has a vertical asymptote at x = 2.
    c. \lim_{x \to 1} f(x) \neq \lim_{x \to 1} f(x) = 2
    d. The function has a vertical asymptote at x = 2.
    e. \lim_{x \to 0^-} |x| = −x
    \lim_{x \to 0^-} \frac{|x|}{x} = 1
    \lim_{x \to 0^+} \frac{|x|}{x} \neq \lim_{x \to 0^+} \frac{|x|}{x}

\text{This agrees well with the values in the table.}

f. \lim_{x \to -1} f(x) = -1
\lim_{x \to -1} f(x) = 5
\lim_{x \to -1} f(x) \neq \lim_{x \to -1} f(x)

Therefore, \lim_{x \to -1} f(x) does not exist.
19. a. \(y = 7\)  
   b. \(y = -5x - 5\)  
   c. \(y = 18x + 9\)  
   d. \(y = -216x + 486\)

20. a. 700,000  
   b. 109,000/h

**Chapter 1 Test, p. 60**

1. \(\lim_{x \to 1} \frac{1}{x - 1} = +\infty \neq \lim_{x \to 1} \frac{1}{x - 1} = -\infty\)

2. \(-13\)

3. a. \(\lim f(x)\) does not exist.  
   b. 1  
   c. 1  
   d. \(x = 1\) and \(x = 2\)

4. a. 1 km/h  
   b. 2 km/h  
   c. \(\sqrt{16 + h - 4}\)

5. \(-31\)

6. \(12\)  
   c. \(\frac{1}{6}\)  
   d. \(\frac{3}{4}\)

7. a. \(-17\)  
   b. 10  
   c. \(\frac{1}{12}\)  
   d. about 7.68

8. \(a = 1, b = -\frac{18}{5}\)

**Chapter 2**

**Review of Prerequisite Skills, pp. 62–63**

1. a. \(a^3\)  
   b. \(-8a^6\)  
   c. \(8ab^6\)  
   d. \(\frac{1}{ab^6}\)  
   e. \(\frac{48a^6}{9b}\)  
   f. \(\frac{2a^3}{2ab}\)

2. a. \(x^2\)  
   b. \(4x^4\)  
   c. \(a^2\)

3. \(-\frac{3}{2}\)  
   b. \(2\)  
   c. \(-\frac{3}{5}\)  
   d. 1

4. a. \(x - 6y - 21 = 0\)  
   b. \(3x - 2y - 4 = 0\)  
   c. \(4x + 3y - 7 = 0\)

5. a. \(2x^2 - 5xy - 3y^2\)  
   b. \(x^3 - 5x^2 + 10x - 8\)  
   c. \(12x^2 + 36x - 21\)  
   d. \(-13x + 42y\)  
   e. \(29x^2 - 2xy + 10y^2\)  
   f. \(-13x^3 - 12x^2y + 4xy^2\)

6. a. \(\frac{15}{2}; x \neq 0, -2\)

7. a. \(\frac{y - 5}{4y^2(y + 2)}\); \(y \neq -2, 0, 5\)  
   b. \(\frac{8}{9}; h \neq -k\)  
   c. \(\frac{2}{(x + y)^2}; x \neq -y, +y\)  
   d. \(11x^2 - 8x + 7\)  
   e. \(2\sqrt{x - 1}; x \neq 0, 1\)  
   f. \(\frac{4x + 7}{(x + 3)(x - 2)}; x \neq -3, 2\)

8. a. \((2k + 3)(2k - 3)\)  
   b. \((x - 4)(x + 8)\)  
   c. \((a + 1)(3a - 7)\)  
   d. \((x^2 + 1)(x + 1)(x - 1)\)  
   e. \((x - y)(x^2 + xy + y^2)\)  
   f. \(\frac{2x + 1}{(r - 1)(r + 2)(r - 2)}\)

9. a. \(-17\)  
   b. 10  
   c. \(\frac{53}{8}\)  
   d. about 7.68

10. a. \(\frac{3\sqrt{2}}{2}\)  
    b. \(\frac{4\sqrt{3} - \sqrt{5}}{3}\)  
    c. \(-\frac{30 + 17\sqrt{2}}{23}\)  
    d. \(-\frac{11 - 4\sqrt{5}}{5}\)

11. a. \(3h + 10\); expression can be used to determine the slope of the secant line between \((2, 8)\) and \((2 + h, f(2 + h))\)  
   b. For \(h = 0.01: 10.03\)  
   c. value represents the slope of the secant line through \((2, 8)\) and \((2.01, 8.1003)\)

**Section 2.1, pp. 73–75**

1. a. \(\{x \in \mathbb{R} | x \neq -2\}\)  
   b. \(\{x \in \mathbb{R} | x \neq 2\}\)  
   c. \(\{x \in \mathbb{R}\}\)  
   d. \(\{x \in \mathbb{R} | x \neq 1\}\)  
   e. \(\{x \in \mathbb{R} | x > 2\}\)  
   f. \(\{x \in \mathbb{R} | x > 2\}\)

2. The derivative of a function represents the slope of the tangent line at a given value of the independent variable or the instantaneous rate of change of the function at a given value of the independent variable.

3. Answers may vary. For example:

4. a. \(5a + 5h - 2; 5h\)  
   b. \(a^2 + 2ah + h^2 + 3a + 3h - 1; 2ah + h^2 + 3h\)  
   c. \(a^2 + 3ah + 3ah^2 + h^3 - 4a - 4h + 1; 3a^2h + 3ah^2 + h^3 - 4h\)  
   d. \(a^2 + 2ah + h^2 + a + h - 6; 2ah + h^2 + h\)  
   e. \(-7a - 7h + 4; -7h\)  
   f. \(4 - 2a - 2h - a^2 - 2ah - h^2; -2h - h^2 - 2ah\)

5. a. \(2\)  
   b. \(\frac{1}{2}\)  
   c. \(-\frac{5}{3}\)  
   d. \(\frac{2}{3}\)  
   e. \(18x^2 - 7\)

6. a. \(-5\)  
   b. \(4x + 4\)  
   c. \(\frac{3}{2}\)  
   d. \(2\sqrt{3x + 2}\)

7. a. \(-7\)  
   b. \(-\frac{2}{(x - 1)^2}\)  
   c. \(6x\)

8. \(f'(0) = -4; f'(1) = 0; f'(2) = 4\)
Answers may vary. For example:

\[ \text{y} = x^2 \]

\[ \text{y}' = 2x \]

\[ \text{y}'' = 2 \]

10. A normal to the graph of a function at a point is a line that is perpendicular to the tangent at the given point:

\[ x + 18y = 25 \]

11. 

12. 

13. 

14. 

Section 2.2, pp. 82–84

1. Answers may vary. For example:

constant function rule: \( \frac{d}{dx}(f(x)) = 0 \)

power rule: \( \frac{d}{dx}(x^n) = nx^{n-1} \)

constant multiple rule: \( \frac{d}{dx}(ax) = a \)

sum rule: \( \frac{d}{dx}(f(x) + g(x)) = \frac{d}{dx}f(x) + \frac{d}{dx}g(x) \)

difference rule: \( \frac{d}{dx}(f(x) - g(x)) = \frac{d}{dx}f(x) - \frac{d}{dx}g(x) \)

2. a. 4  
   d. \( \frac{1}{3\sqrt{x^2}} \)

b. \( 3x^2 - 2x \)  
   e. \( \frac{x^3}{4} \)

c. \(-2x + 5\)  
   f. \(-3x^{-4}\)

3. a. \( 4x + 11 \)  
   d. \( x^4 + x^2 - x \)

b. \( 6x^2 + 10x - 4 \)  
   e. \( 40x^7 \)

c. \( 4x^3 - 6x^2 \)  
   f. \( 2x^3 - \frac{3}{2} \)

4. a. \( 5x^2 \)  

b. \( -2x^{-\frac{1}{2}} + 6x^{-2} \)  

c. \( \frac{18}{x^4} - \frac{4}{x^3} \)  

d. \( -18x^{-3} + \frac{3}{2} \frac{1}{x^2} \)

5. a. \(-4t + 7\)  
   b. \( 5 - t^2 \)  
   c. \( 2t - 6 \)

6. a. 47.75  
   b. \( \frac{11}{24} \)

7. a. 12  
   c. \(-\frac{1}{2}\)

b. 5  
   d. 12

8. a. 9  
   c. 4

b. \( \frac{1}{2} \)  
   d. \(-7\)

9. a. \( 6x - y - 4 = 0 \)  
   b. \( 18x - y + 25 = 0 \)

b. \( 9x - 2y = 9 = 0 \)

d. \( x + y - 3 = 0 \)

e. \( 7x - 2y - 28 = 0 \)

f. \( 5x - 6y - 11 = 0 \)

10. A normal to the graph of a function at a point is a line that is perpendicular to the tangent at the given point:

\[ x + 18y = 25 = 0 \]

11. 

12. 

13. 

14. 

(3, -8)

20. \( 2x + y + 1 = 0 \) and \( 6x - y - 9 = 0 \)

15. \( (2, 10) \) and \( (-2, -6) \)

16. \( y = \frac{1}{5} x^5 - 10x \), slope is 6

\[ \frac{dy}{dx} = x^4 - 10 = 6 \]

\[ x^4 = 16 \]

\[ x^2 = 4 \text{ or } x^2 = -4 \]

\[ x = \pm 2 \text{ non-real} \]

Tangents with slope 6 are at the points \( (2, -\frac{68}{5}) \) and \( (-2, \frac{68}{5}) \).

17. a. \( y - 3 = 0; 16x - y - 29 = 0 \)

b. \( 20x - y - 47 = 0; 4x + y - 1 = 0 \)

18. 

19. a. 49.9 km  
   b. 0.12 km/m
20. a. 34.3 m/s  
   b. 39.2 m/s  
   c. 54.2 m/s  
21. 0.29 min and 1.71 min  
22. −20 m/s  
23. (1, −3) and (−1, −3)  

24. B(0, 0)  

25. a. i. \( \left( \frac{1}{5}, \frac{1}{3} \right) \)  
   ii. \( \left( \frac{1}{4}, -\frac{13}{4} \right) \)  
   iii. \( \frac{103}{27} \) and (5, −47)  
   b. At these points, the slopes of the tangents are zero, meaning that the rate of change of the value of the function with respect to the domain is zero. These points are also local maximum and minimum points.  
26. \( \sqrt{x} + \sqrt{y} = 1 \)  
P(a, b) is on the curve; therefore, 
\( a \geq 0, b \geq 0, \sqrt{y} = 1 - \sqrt{x} \)  
y = 1 − 2√x + x  
\( \frac{dy}{dx} = -\frac{1}{2}(2x^{-\frac{1}{2}} + 1) \)  
At \( x = a \). Slope is 
\( -\frac{1}{\sqrt{a}} + 1 = -\frac{1 + \sqrt{a}}{\sqrt{a}} \)  
But, \( \sqrt{a} + \sqrt{b} = 1 \)  
\( -\sqrt{b} = \sqrt{a} - 1 \)  
Therefore, slope is \( -\frac{\sqrt{b}}{\sqrt{a}} = -\frac{\sqrt{b}}{a} \)  

27. The x-intercept is \( 1 - \frac{1}{n} \), as \( n \to \infty, \frac{1}{n} \to 0 \), and the x-intercept approaches 1. As \( n \to \infty \), the slope of the tangent at \( (1, 1) \) increase without bound, and the tangent approaches a vertical line having equation \( x = 1 \).

28. a. \( f'(x) = \begin{cases} 
               2x, & \text{if } x < 3 \\
               1, & \text{if } x \geq 3 
             \end{cases} 
\)  
f'(3) does not exist.

Section 2.3, pp. 90–91  
1. a. \( 2x - 4 \)  
b. \( 6x^2 - 2x \)  
c. \( 12x - 17 \)  
d. \( 45x^4 - 80x^3 + 2x - 2 \)  
e. \( -8t^3 + 2t \)  
f. \( \frac{6}{(x + 3)^2} \)  
2. a. \( (5x + 1)^3 + 15(5x + 1)^2(x - 4) \)  
b. \( 15x^2(3x^2 + 4)(3 + x^3)^4 \\
    + 60x(3 + x^3)^3 \)  
c. \( -8[1 - x^3]^3(2x + 6)^3 + 6(1 - x^3)^2(2x + 6)^2 \)  
d. \( 6(x^2 - 9)^3(2x - 1)^2 + 80(x^2 - 9)^2(2x - 1)^3 \)  
3. It is not appropriate or necessary to use the product rule when one of the factors is a constant or when it would be easier to first determine the product of the factors and then use other rules to determine the derivative. For example, it would not be best to use the product rule for \( f(x) = 3(x^2 + 1) \) or \( g(x) = (x + 1)(x - 1) \).  
4. \( F'(x) = [b(x) + c'(x)] + [b'(x)]c(x) \)  
5. a. 9  
b. −4  
c. 22  
d. −9  
e. 671  
6. 10x + y = 8 = 0  
7. a. (14, −450)  
b. (−1, 0)  
8. a. \( 3(x + 1)^2(x + 4)(x - 3)^2 + (x + 1)^3(1)(x - 3)^2 + (x + 1)^3(x + 4)(2)(x - 3) \)  
b. \( 2x(3x^2 + 4)(3 - x^3)^4 + x^2[2(3x^2 + 4)(6x)](3 - x^3)^4 + x^2[3x^2 + 4]^3 \times [4(3 - x^3)^3(3x^2)] \)  
9. −4.841 L/h  
10. −30; Determine the point of tangency, and then find the negative reciprocal of the slope of the tangent. Use this information to find the equation of the normal.
11. a. \( f(x) = g_1(x)g_2(x)g_3(x) \ldots g_n(x) \)
\( g_n - f(x)g_n(x) \)
\( g_1(x)g_2(x)g_3(x) \ldots g_n - f(x)g_n(x) \)
\( g_1(x)g_2(x)g_3(x) \ldots g_n - f(x)g_n(x) \)
\( g_1(x)g_2(x)g_3(x) \ldots g_n - f(x)g_n(x) \)
\( g_1(x)g_2(x)g_3(x) \ldots g_n - f(x)g_n(x) \)
\( g_1(x)g_2(x)g_3(x) \ldots g_n - f(x)g_n(x) \)
\( g_n - f(x)g_n(x) \)
b. \( \frac{2}{x+1} \)
12. \( y = 3x^2 + 6x - 5 \)
13. \( y = \frac{16}{x^2} - 1 \)
\( dy \)
\( dx = -\frac{32}{x^3} \)
Slope of this line is 4.
\( \frac{32}{x^3} = 4 \)
x = -2
y = 3
Point is at \((-2, 3)\).
Find intersection of line and curve:
y = 4x + 11
Substitute,
\( 4x + 11 = \frac{16}{x^2} - 1 \)
\( 4x^3 + 11x^2 - 16 - x^2 \) or
\( 4x^3 + 12x^2 - 16 = 0 \)
Let \( x = -2 \)
\( RS = 4(-2) + 12(-2)^2 - 16 \)
\( 0 \)
Since \( x = -2 \) satisfies the equation, therefore it is a solution.
When \( x = -2, y = 4(-2) + 11 = 3. \)
Intersection point is \((-2, 3)\). Therefore, the line is tangent to the curve.

14. Mid-Chapter Review, pp. 92–93
1. a. \( f(x) \) is quadratic; \( f'(x) \) is linear.
2. a. 6
b. 4x
c. \( \frac{-5}{(x+5)^2} \)
d. \( \frac{1}{2\sqrt{x-2}} \)
3. a. \( y = -2x + 2 \)
b. \( y = 2x^3 \)
c. \( \frac{5}{\sqrt{x}} \)
d. \( 5 - \frac{6}{x^2} \)
e. 24t + 22
f. \( \frac{1}{x^2} \)
4. a. 24x^3
b. \( \frac{5}{\sqrt{x}} \)
c. \( \frac{6}{x} \)
d. \( 5 - \frac{6}{x^2} \)
e. 24t + 22
f. \( \frac{1}{x^2} \)
5. a. \( y = x - \frac{3}{8} \)
b. \( -6x^2 + 8x + 5 \)
c. \( \frac{10}{x^2} + \frac{9}{x} \)
d. \( \frac{1}{2x^3} + \frac{1}{3x^2} \)
e. \( \frac{4}{x^2} + 5 \)
f. \( \frac{1}{x^2} \)
6. a. \( y = \frac{1}{3}x^3 \)
b. \( y = \frac{1}{3}x^3 + 297 \)
c. \( y = -12x^2 + 297 \)
d. \( y = -12x + 297 \)
7. a. \( y = 7 \)
b. \( y = \frac{1}{3}x^3 \)
c. \( y = \frac{1}{3}x^3 + 297 \)
8. a. 48x^3 - 81x^2 + 40x - 45
b. \( -36x^2 - 50x + 39 \)
c. \( 24x^3 + 24x^2 - 78x - 36 \)
d. \( -162x^3 + 216x^2 - 72x \)
9. \( 76x - y - 28 = 0 \)
10. \( (3, 8) \)
11. \( 10x - 8 \)
12. a. \( 500 \frac{9}{L}{L}/min \)
b. \( 200 \frac{27}{L}/min \)
c. \( -200 \frac{27}{L}/min \)
13. a. \( \frac{1900}{3} \pi \ cm^3/cm \)
b. \( 256 \pi \ cm^3/cm \)
14. This statement is always true. A cubic polynomial function will have the form
\( f(x) = ax^3 + bx^2 + cx + d, a \neq 0 \).
So, the derivative of this cubic is
\( f'(x) = 3ax^2 + 2bx + c \) and since
\( 3a \neq 0 \), this derivative is a quadratic polynomial function. For example, if
\( f(x) = x^3 + x^2 + 1 \), we get
\( f'(x) = 3x^2 + 2x \), and if
\( f(x) = 2x^3 + 3x^2 + 6x + 2 \), we get
\( f'(x) = 6x^2 + 6x + 6 \).
15. \( y = \frac{x^2 + 3b}{x^3 + 4b}, a, b \in I \)
Simplifying,
y = \( x^{a+b} - (a+b) = x^a + 4b \)
Then,
y' \( (a + 4b)^{a+b+1} \)
16. a. \( -188 \)
b. \( f'(3) \) is the slope of the tangent line
to \( f(x) \) at \( x = 3 \) and the rate of change in the value of \( f(x) \) with
respect to \( x \) at \( x = 3 \).
Section 2.4, pp. 97–98

1. For \(x, a, b\) real numbers, \(x^a x^b = x^{a+b}\)
   
   Also, 
   \(x^a x^b = x^{a-b}, x \neq 0\)

2. In the previous problem, all of these rational examples could be differentiated via the power rule after a minor algebraic simplification. A second approach would be to rewrite a rational example 
   \(h(x) = \frac{f(x)}{g(x)}\)
   
   using the exponent rules as 
   \(h(x) = f(x) g(x)^{-1}\), and then apply the product rule for differentiation (together with the power of a function rule) to find \(h'(x)\). A third (an perhaps easiest) approach would be to just apply the quotient rule to find \(h'(x)\).

3. If \(f(x)\) and \(g(x)\) are two differentiable functions of \(x\), and
   \(h(x) = (f \circ g)(x) = f(g(x))\)
   is the composition of these two functions, then \(h'(x) = f'(g(x)) \cdot g'(x)\).

   This is known as the “chain rule” for differentiation of composite functions. For example, if \(f(x) = x^{10}\) and \(g(x) = x^2 + 3x + 5\), then \(h(x) = (x^{10})^2\), and so \(h'(x) = 2 f'(g(x)) g'(x) = 10 x^2 (x^2 + 3x + 5)^2 (2x + 3)\)

   As another example, if \(f(x) = x^2\) and \(g(x) = x^2 + 1\), then \(h(x) = (x^2 + 1)^2\), and so \(h'(x) = 2 (x^2 + 1)^{-1} (2x)\).

4. a. \(8(2x + 3)^3\)
   b. \(6x(x^2 - 4)^2\)
   c. \(4(2x^2 + 3x - 5)^3 (4x + 3)\)
   d. \(-6x \pi^2 \cdot x^2\)
   e. \(\frac{x}{\sqrt{x^2 + 3}} + 3\)
   f. \(\frac{-10x}{(x^2 - 16)^6}\)

5. a. \(-2x^3\) \(\frac{6}{x^2}\)
   b. \(a + 1 - 1\)
   c. \((x^2 - 4)^{-1}\)
   d. \(3(9 - x^2)^{-1}\)
   e. \((5x^2 + x)^{-1}\)
   f. \((x^2 + x + 1)^{-1}\)

6. \(h(-1) = -4; h'(-1) = 35\)

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**Section 2.5, pp. 105–106**

1. a. 0
   b. 0
   c. \(\sqrt{x^2 - 1}\)
   d. \(\sqrt{15}\)
   e. \(\sqrt{x^2 + 1}\)
   f. \(-1\)

2. a. \(f \circ g = x, \quad (g \circ f) = |x|, \quad \{x \neq 0\}, \{x \in \mathbb{R}\}:\ not\ equal\)
   b. \((f \circ g) = \frac{1}{(x^2 + 1)^2}\)
   c. \(f \circ g = \frac{1}{x^2 + 2}, \quad \{x > -2\}, \left\{ \left\{x \leqslant -\frac{1}{2} f, x > 0\right\}\right\}\)
   d. \(f \circ g = \frac{1}{\sqrt{x + 2}}\)

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**Answers 633**
8. a. \((x + 4)^2(x - 3)^3(9x + 15)\)
b. \(6x(x^2 + 3)^2(x^3 + 3)\)
   \((2x^3 + 3x + 3)\)
c. \(-2x^2 + 6x + 2\)
   \((x^2 + 1)^2\)
d. \(15x^2(3x - 5)(x - 1)\)
e. \(4x^3(1 - 4x^2)(1 - 10x^2)\)
f. \(48x(x^2 + 3)^3\)
9. a. \(91\)
b. \(-5\sqrt{2}\)
10. \(x = 0\) or \(x = 1\)
11. \(1\)
12. \(60x - y = 119 = 0\)
13. a. 52  b. 54  c. 320  d. 78
14. \(-6\)
15. 2222 L/min
16. 2.75 m/s
17. a. \(p'(x)q(x)r(x) + p(x)q'(x)r(x)\)
   \(\frac{dy}{dx} + p(x)q(x)r'(x)\)
b. \(-344\)
18. \(\frac{dy}{dx} = 3(x^2 + x - 2)^2(2x + 1)\)
   At the point (1, 3), slope of the tangent will be \(3(1 - 1 - 2)^2(2 + 1) = 0\).
   Equation of the tangent at (1, 3) is
   \[y = 3(x + 2)^3 + 3 = 3\]
   \[(x + 2)^3(x - 1)^3 = 0\]
   \(x = -2\) or \(x = 1\)
   Since \(-2\) and 1 are both triple roots, the line with equation \(y = 3\) is also a tangent at \((-2, 3)\).
19. \(\frac{-2(x^2 + 3x - 1)(1 - x)^3}{(1 + x)^3}\)

**Review Exercise, pp. 110–113**

1. To find the derivative \(f'(x)\), the limit
   \(f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}\)
   must be computed, provided it exists. If this limit does not exist, then the derivative of \(f(x)\) does not exist at this particular value of \(x\). As an alternative to this limit, we could also find \(f'(x)\) from the definition by computing the equivalent limit
   \(f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}\). These
   two limits are seen to be equivalent by substituting \(z = x + h\).
2. a. \(4x - 5\)
   c. \(\frac{4}{(4 - x)^3}\)
b. \(\frac{1}{2\sqrt{x} - 6}\)
3. a. \(2x - 5\)
   c. \(-\frac{28}{3x^2}\)
b. \(\frac{3}{4x^2}\)
e. \(\frac{2x}{(x^1 + 5)^2}\)
f. \(\frac{7x + 2}{\sqrt{7x^2 + 4x + 1}}\)
4. a. \(2 + \frac{2}{x}\)
b. \(\frac{\sqrt{x}}{2}(x^2 - 3)\)
e. \(-\frac{5}{(3x - 5)^2}\)
f. \(\frac{3}{2\sqrt{x}} - 1\)
5. a. \(20x^3(2x - 5)^5(x - 1)\)
b. \(\frac{2x}{\sqrt{x^2 + 1}} + \frac{x^2 + 1}{(2x - 5)^3(2x + 23)}\)
c. \(\frac{x + 1}{(x + 1)^3}\)
d. \(\frac{31800 - 1}{10x^2 - 1}\)
e. \(\frac{6(1 - x^2)^2(x^2 + 6x + 1)}{(6 + 2x)^4}\)
f. \(\frac{61}{24}\)
6. a. \(f'(x^2) \times 2x\)
b. \(2\sqrt{f'(x)} + 2f(x)\)
7. a. \(-\frac{184}{9}\)
b. \(-\frac{25}{289}\)
e. \(-\frac{8}{5}\)
8. \(-\frac{2}{3}\)
9. \(x = 2 \pm 2\sqrt{2}\)
   \(x = 5, x = -1\)
10. a. \(x = 0, x = \pm 2\)
   i. \(x = 0, x = \pm 1, x = \pm \frac{\sqrt{3}}{3}\)
   b. \(x = 0, x = \pm 1, x = \pm \frac{\sqrt{3}}{3}\)
11. a. $160x - y + 16 = 0$
   b. $60x + y - 61 = 0$
12. $5x - y - 7 = 0$
13. (2, 8); $b = -8$
14. a. 
   b. $y = 0, y = 6.36, y = -6.36$
   c. $(0, 0), \left(3\sqrt{2}, \frac{9\sqrt{2}}{2}\right)$, 
      $\left(-3\sqrt{2}, -\frac{9\sqrt{2}}{2}\right)$
   d. $-14$
15. a. $\sqrt{50}$
   b. 1
16. a. When $t = 10, 9$; when $t = 15, 19$
   b. At $t = 10$, the number of words memorized is increasing by 1.7 words/min. At $t = 15$, the number of words memorized is increasing by 2.325 words/min.
17. a. $\frac{30t}{(9 + t)^2}$
   b. No; since $t > 0$, the derivative is always positive, meaning that the rate of change in the cashier’s productivity is always increasing. However, these increases must be small, since, according to the model, the cashier’s productivity can never exceed 20.
18. a. $x^2 + 40$
   b. 6 gloves/week
19. a. $750 - \frac{x}{3} - 2x^2$
   b. $\$546.67
20. $-\frac{5}{4}$
21. a. $B(0) = 500, B'(30) = 320$
   b. $B'(0) = 0, B'(30) = -12$
   c. $B(0) = $ blood sugar level with no insulin
      $B(30) = $ blood sugar level with 30 mg of insulin
      $B'(0) = $ rate of change in blood sugar level with no insulin
      $B'(30) = $ rate of change in blood sugar level with 30 mg of insulin
   d. $B'(50) = -20, B(50) = 0$
      $B'(50) = -20$ means that the patient’s blood sugar level is decreasing at 20 units/mg of insulin 1 h after 50 mg of insulin is injected.
      $B(50) = 0$ means that the patient’s blood sugar level is zero 1 h after 50 mg of insulin is injected. These values are not logical because a person’s blood sugar level can never reach zero and continue to decrease.
22. a. $f(x)$ is not differentiable at $x = 1$ because it is not defined there (vertical asymptote at $x = 1$).
   b. $g(x)$ is not differentiable at $x = 1$ because it is not defined there (hole at $x = 1$).
   c. The graph has a cusp at $(2, 0)$ but is differentiable at $x = 1$.
   d. The graph has a corner at $x = 1$, so $m(t)$ is not differentiable at $x = 1$.
23. a. $f(x)$ is not defined at $x = 0$ and $x = 0.25$. The graph has vertical asymptotes at $x = 0$ and $x = 0.25$.
   Therefore, $f(x)$ is not differentiable at $x = 0$ and $x = 0.25$.
   b. $f(x)$ is not defined at $x = 3$ and $x = -3$. At $x = -3$, the graph has a vertical asymptote and at $x = 3$ it has a hole. Therefore, $f(x)$ is not differentiable at $x = 3$ and $x = -3$.
   c. $f(x)$ is not defined for $1 < x < 6$.
   Therefore, $f(x)$ is not differentiable for $1 < x < 6$.
24. $\frac{25}{(t + 1)^2}$
25. Answers may vary. For example:
   $f(x) = 2x + 3$
   $y = \frac{1}{(2x + 3)(0) - (1)(2)}$
   $y' = -\frac{2}{(2x + 3)^2}$
   $f(x) = 5x + 10$
   $y = \frac{1}{(5x + 10)(0) - (1)(5)}$
   $y' = -\frac{5}{(5x + 10)^2}$
Rule: If $f(x) = ax + b$ and $y = \frac{1}{f(x)}$,
then $y' = -\frac{a}{(ax + b)^2}$
   $y' = \lim_{h \to 0} \frac{1}{h}[\frac{a(x+h) + b}{ax+b} - \frac{1}{ax+b}]
   = \lim_{h \to 0} \frac{1}{h}[ax + b - (a(x+h) + b)]
   = \lim_{h \to 0} \frac{1}{h} [a(x+h) + b][ax+b]$
3. \(1 - 2x\)  
4. a. \(x^2 + 15x^{-6}\)  
   b. \(60(2x - 9)^4\)  
   c. \(-x^2 + \frac{1}{\sqrt{x}} + 2x^{-\frac{3}{2}}\)  
   \[5(x^2 + 6)^4(3x^2 + 8x - 18)\]  
   d. \(\frac{5}{(3x + 4)^6}\)  
   e. \(2(6x^2 - 7)^{-\frac{3}{2}}(8x^2 - 7)\)  
   f. \(4x^5 - 18x + 8\)  
5. 14  
6. \(-\frac{40}{3}\)  
7. \(60x + y - 61 = 0\)  
8. \(\frac{75}{32}\) ppm/year  
9. \((-\frac{1}{4}, \frac{1}{256})\)  
10. \((-\frac{1}{3}, \frac{32}{27})\), (1, 0)  
11. \(a = 1, b = -1\)

**Chapter 3**  
Review of Prerequisite Skills, pp. 116–117

1. a.  
   ![Graph](image1.png)  
2. a. \(x = \frac{14}{5}\)  
   b. \(x = -13\)  
   c. \(t = 3\) or \(t = 1\)  
   d. \(t = -\frac{1}{2}\) or \(t = 3\)  
   e. \(t = 2\) or \(t = 6\)  
   f. \(x = 0\) or \(x = -3\) or \(x = 1\)  
   g. \(x = 0\) or \(x = 4\)  
   h. \(t = -3\) or \(t = \frac{1}{2}\) or \(t = -\frac{1}{2}\)  
   i. \(t = \pm\frac{3}{2}\) or \(t = \pm 1\)  
3. a. \(x > 3\)  
   b. \(x < 0\) or \(x > 3\)  
   c. \(0 < x < 4\)  
4. a. \(25\) cm\(^2\)  
   b. \(48\) cm\(^2\)  
   c. \(49\pi\) cm\(^2\)  
   d. \(36\pi\) cm\(^2\)  
5. a. \(SA = 56\pi\) cm\(^2\), \(V = 48\pi\) cm\(^3\)  
   b. \(h = 6\) cm, \(SA = 80\pi\) cm\(^2\)  
   c. \(r = 6\) cm, \(SA = 144\pi\) cm\(^2\)  
   d. \(h = 7\) cm, \(V = 175\pi\) cm\(^3\)  
6. a. \(SA = 54\) cm\(^2\), \(V = 27\) cm\(^3\)  
   b. \(SA = 30\) cm\(^2\), \(V = 5\sqrt{5}\) cm\(^3\)  
   c. \(SA = 72\) cm\(^2\), \(V = 24\sqrt{3}\) cm\(^3\)  
   d. \(SA = 24\pi\) cm\(^2\), \(V = 8\pi\) cm\(^3\)  
7. a. \((3, \infty)\)  
   b. \((-\infty, -2]\)  
   c. \((-\infty, 0]\)  
   d. \([-5, \infty)\)  
   e. \((-2, 8]\)  
   f. \((-4, 4]\)  
8. a. \(\{x \in \mathbb{R} \mid x > 5\}\)  
   b. \(\{x \in \mathbb{R} \mid x < 1\}\)  
   c. \(\{x \in \mathbb{R}\}\)  
   d. \(\{x \in \mathbb{R} \mid -10 \leq x \leq 12\}\)  
   e. \(\{x \in \mathbb{R} \mid -1 < x < 3\}\)  
   f. \(\{x \in \mathbb{R} \mid 2 \leq x < 20\}\)  
9. a.  
   ![Graph](image2.png)  
   The function has a minimum value of \(-5\) and no maximum value.  
   b.  
   ![Graph](image3.png)  
   The function has a maximum value of \(25\) and no minimum value.  
   c.  
   ![Graph](image4.png)  
   The function has a minimum value of \(7\) and no maximum value.
2. a. \( y'' = 90x^8 + 90x^4 \)
   b. \( f''(x) = -\frac{1}{4}x^{-\frac{3}{2}} \)
   c. \( y'' = 2 \)
   d. \( h''(x) = 36x^2 - 24x - 6 \)
   e. \( y'' = \frac{3}{\sqrt{x}} - \frac{6}{x^3} \)
   f. \( f''(x) = -\frac{4}{(x + 1)^3} \)
   g. \( y'' = 2 + \frac{6}{x^3} \)
   h. \( g''(x) = -\frac{9}{4(3x - 6)^2} \)
   i. \( y'' = 48x + 96 \)
   j. \( h''(x) = \frac{10}{9x^2} \)

3. a. \( v(t) = 10t - 3 \)
   b. \( a(t) = 10 \)
   c. \( v(t) = 6t^2 + 36 \)
   d. \( a(t) = 12t \)
   e. \( v(t) = 1 - 6t^{-2} \)
   f. \( a(t) = 12t^{-3} \)
   g. \( v(t) = 2(t - 3) \)
   h. \( a(t) = 2 \)

4. a. \( i. t = 3 \)
   b. \( i. t = 3, t = 7 \)

5. a. \( v(t) = t^2 - 4t + 3 \)
   b. \( a(t) = 2t - 4 \)
   c. \( t > 3 \)

6. a. For \( t = 1 \), moving in a positive direction.
   b. For \( t = 4 \), moving in a negative direction.
   c. For \( t = 1 \), moving in a positive direction.
   d. For \( t = 4 \), moving in a negative direction.

7. a. \( v(t) = 2t - 6 \)
   b. \( t = 3 \)

9. a. \( v(5) = 3 \) m/s
   b. \( a(5) = 2 \) m/s^2

10. a. \( v(t) = \frac{35}{2}t^2 - \frac{7}{2}t^2 \)
    b. \( a(t) = \frac{105}{4}t^2 - \frac{35}{4}t^2 \)

Section 3.1, pp. 127–129

1. At \( t = 1 \), the velocity is positive; this means that the object is moving in whatever is the positive direction for the scenario. At \( t = 5 \), the velocity is negative; this means that the object is moving in whatever is the negative direction for the scenario.
14. $t = 1$ s; away
15. a. $s(t) = kt^2 + (6k^2 - 10k)t + 2k$
   $v(t) = 2kt + (6k^2 - 10k)$
   $a(t) = 2k + 0$
   $= 2k$
   Since $k \neq 0$ and $k \in \mathbb{R}$, then $a(t) = 2k \neq 0$ and an element of the real numbers. Therefore, the acceleration is constant.

b. $t = 5 - 3k, -9k^3 + 30k^2 - 23k.$
16. a. The acceleration is continuous at $t = 0$ if $\lim_{t \to 0} a(t) = a(0)$.
   For $t \geq 0$,
   
   
   $$s(t) = \frac{t^3}{t^2 + 1}$$
   
   and
   
   $$v(t) = \frac{3t^2(t^2 + 1) - 2t(t^2)}{(t^2 + 1)^2}$$
   
   and
   
   $$a(t) = \frac{(4t^3 + 6t)(t^2 + 1)^2}{(t^2 + 1)^2} - 2(t^2 + 1)(2t)(t^4 + 3t^2)}{(t^2 + 1)^2}$$
   
   $$= \frac{(4t^3 + 6t)(t^2 + 1) - 4t(t^4 + 3t^2)}{(t^2 + 1)^3}$$
   
   $$= \frac{4t^5 + 6t^3 + 4t^3}{(t^2 + 1)^3}$$
   
   $$= \frac{6t - 4t^3 - 12t^3}{(t^2 + 1)^3}$$
   
   $$= \frac{-2t^3 + 6t}{(t^2 + 1)^3}$$
   
   Therefore,
   
   $$a(t) = \begin{cases} 
   0, & \text{if } t < 0 \\
   \frac{-2t^3 + 6t}{(t^2 + 1)^3}, & \text{if } t \geq 0 
   \end{cases}$$
   
   and
   
   $$v(t) = \begin{cases} 
   0, & \text{if } t < 0 \\
   \frac{t^4 + 3t^2}{(t^2 + 1)^2}, & \text{if } t \geq 0 
   \end{cases}$$
   
   $$\lim_{t \to 0} a(t) = 0,$$
   
   $$\lim_{t \to 0} a(t) = 0$$
   
   Thus, $\lim_{t \to 0} a(t) = 0$.
   
   Also, $a(0) = 0$
   
   Therefore, $\lim_{t \to 0} a(t) = a(0)$.
   
   Thus, the acceleration is continuous at $t = 0$.

b. velocity approaches 1, acceleration approaches 0
17. $v = \sqrt{b^2 + 2gx}$
   $\frac{dv}{dt} = \frac{1}{2}(b^2 + 2gx)^{-\frac{1}{2}} \times \left( 0 + 2g \frac{dx}{dt} \right)$
   
   $$a = \frac{1}{2}v \times 2gv$$
   
   $$a = g$$
   
   Since $g$ is a constant, $a$ is a constant, as required.

Note: $\frac{dx}{dt} = v$

$$\frac{dv}{dt} = a$$

18. $F = m\frac{dv}{dt} \div \sqrt{1 - (\frac{v}{c})^2}$

Using the quotient rule,

$$m\frac{dv}{dt} \div \left(1 - \frac{v^2}{c^2}\right)$$

$$= \frac{1}{\left[1 - \frac{v^2}{c^2}\right]^2} \times \left(1 - \frac{v^2}{c^2}\right)$$

Since $\frac{dv}{dt} = a$,

$$= m\left[1 - \frac{v^2}{c^2}\right] \div \left[\left(1 - \frac{v^2}{c^2}\right)^3 + \left(1 - \frac{v^2}{c^2}\right)\right]$$

$$= m\frac{ac^2}{c^2(1 - \frac{v}{c})^2}$$

$$= m\frac{ac}{(1 - \frac{v}{c})^2},$$

as required.

Section 3.2, pp. 135–138

1. a. The algorithm can be used; the function is continuous.
   b. The algorithm cannot be used; the function is discontinuous at $x = 2$.
   c. The algorithm cannot be used; the function is discontinuous at $x = 2$.
   d. The algorithm can be used; the function is continuous on the given domain.

2. a. max: 8, min: −12
   b. max: 30, min: −5
   c. max: 100, min: −100
   d. max: 30, min: −20

c. min is −4 at $x = -1, 2$, max is 0 at $x = 0, 3$

d. max is 0 at $x = 0$, min is −20 at $x = −1$
11. Absolute max value: Compare all local maxima and values of \( f(a) \) and \( f(b) \) when the domain of \( f(x) = a \leq x \leq b \). The one with the highest value is the absolute maximum.

Absolute min: We need to consider all local minima and the value of \( f(a) \) and \( f(b) \) when the domain of \( f(x) = a \leq x \leq b \). Compare them, and the one with the lowest value is the absolute minimum.

You need to check the endpoints because they are not necessarily critical points.

12. a. 
   \[ y' = -\frac{\sqrt{3}}{3} x \]
   b. \(-2 \leq x \leq 4\)
   c. increasing: \(-2 \leq x < 0\)
   decreasing: \(0 < x < 2\)

13. Absolute max: Compare all local maxima and values of \( f(a) \) and \( f(b) \) when the domain of \( f(x) = a \leq x \leq b \). The one with the highest value is the absolute maximum.

Absolute min: We need to consider all local minima and the value of \( f(a) \) and \( f(b) \) when the domain of \( f(x) = a \leq x \leq b \). Compare them, and the one with the lowest value is the absolute minimum.

You need to check the endpoints because they are not necessarily critical points.

14. 245 units
15. 300 units

**Mid-Chapter Review, pp. 139–140**

1. a. \( h''(x) = 36x^2 - 24x - 6 \)
   b. \( f''(x) = 48x - 120 \)
   c. \( y'' = \frac{30}{(x + 3)^3} \)
   d. \( g''(x) = -\frac{x^2}{(x^2 + 1)^3} + \frac{1}{(x^2 + 1)^2} \)

2. a. 108 m
   b. \(-45 \text{ m/s} \)
   c. \(-18 \text{ m/s}^2 \)

3. a. 6 m/s
   b. \( t = 0.61 \text{ s} \)
   c. \( t = 1.50 \text{ s} \)
   d. \(-8.67 \text{ m/s} \)
   e. \(-9.8 \text{ m/s}^2, -9.8 \text{ m/s}^2 \)

4. a. Velocity is 0 m/s
   Acceleration is 10 m/s^2
   b. Object is stationary at time \( t = \frac{1}{3} \text{ s} \) and \( t = 2 \text{ s} \)
   Before \( t = \frac{1}{3} \), \( v(t) \) is positive and therefore the object is moving to the right.

Between \( t = \frac{1}{3} \) and \( t = 2 \), \( v(t) \) is negative and therefore the object is moving to the left.

After \( t = 2 \), \( v(t) \) is positive and therefore the object is moving to the right.

5. a. min value is 1 when \( x = 0 \),
   max value is 21 when \( x = 2 \)
   b. min value is 0 when \( x = -2 \),
   max value is 25 when \( x = 3 \)
   c. min value is 0 when \( x = 1 \),
   max value is 0.38 when \( x = 3\)

6. \( 30^\circ \text{C} \)
7. a. 105
   b. 3
   c. \(-6 \)
   d. \(-\frac{202}{27} \)
   e. \(-1.7 \text{ m/s}^2 \)
8. a. 189 m/s
   b. 27 s
   c. 2916 m
   d. 6.2 m/s^2
9. 10 m/s
10. 16 m; 4 s
11. a. \( 0 \leq t \leq 4.31 \)
    b. 2.14 s
    c. 22.95 m

**Section 3.3, pp. 145–147**

1. 25 cm by 25 cm
2. If the perimeter is fixed, then the figure will be a square.
3. 150 m by 300 m
4. height 8.8 cm, length 82.4 cm, and width 22.4 cm
5. 110 cm by 110 cm
6. 8 m by 8 m
7. 125 m by 166.67 m
8. 4 m by 6 m by 6 m
9. base 10 cm by 10 cm, height 10 cm
10. 100 square units when \( 5 \sqrt{2} \)
11. a. \( r = 5.42 \), \( h = 10.84 \)
    b. \( h = \frac{1}{r} \); yes
12. a. \( 15 \text{ cm}^2 \) when \( W = 2.5 \text{ cm} \) and \( L = 6 \text{ cm} \)
    b. \( 30 \text{ cm}^2 \) when \( W = 4 \text{ cm} \) and \( L = 7.5 \text{ cm} \)
    c. The largest area occurs when the length and width are each equal to one-half of the sides adjacent to the right angle.
13. a. base is 20 cm and each side is 20 cm
    b. approximately \( 260,000 \text{ cm}^3 \)
14. a. triangle side length 0.96 cm, rectangle 0.96 cm by 1.09 cm
   b. Yes. All the wood would be used for the outer frame.
15. 0.36 h after the first train left the station
16. 1:02 p.m.; 3 km
17. 
   \[ a^2 - b^2 = L \]
   \[ \frac{a^2 - b^2}{a^2} = \frac{W}{2ab} \]
   \[ W = \frac{2ab}{a^2 - b^2}(a^2 - b^2 - L) \]
   \[ A = LW = \frac{2ab}{a^2 - b^2}[a^2L - b^2L - L^2] \]
   Let \( \frac{dA}{dL} = a^2 - b^2 - 2L = 0 \),
   \[ L = \frac{a^2 - b^2}{2} \]
   and
   \[ W = \frac{2ab}{a^2 - b^2} \left[ a^2 - b^2 - \frac{a^2 - b^2}{2} \right] = \frac{ab}{a} \]
   The hypothesis is proven.
18. Let the height be \( h \) and the radius \( r \).
   Then, \( \pi r^2h = k \), \( h = \frac{k}{\pi r^2} \).
   Let \( M \) represent the amount of material,
   \[ M = 2\pi r^2 + 2\pi rh \]
   \[ = 2\pi r^2 + 2\pi \left( \frac{k}{\pi r^2} \right) \]
   \[ = 2\pi r^2 + \frac{2k}{r} \]
   \[ 0 \leq r \leq \infty \]

Using the max min Algorithm,
\[ \frac{dM}{dr} = 4\pi r - \frac{2k}{r^2} \]
Let \( \frac{dM}{dr} = 0, r^3 = \frac{k}{4\pi}, r \neq 0 \) or
\[ r = \left( \frac{k}{2\pi} \right)^{\frac{1}{3}}. \]
When \( r \to 0, M \to \infty \)
\( r \to \infty, M \to \infty \)
\[ r = \left( \frac{k}{2\pi} \right)^{\frac{1}{3}} \]
\[ d = 2\left( \frac{k}{2\pi} \right)^{\frac{1}{3}} \]
\[ h = \frac{k}{\pi \left( \frac{k}{2\pi} \right)^{\frac{1}{3}}} = \frac{\pi}{k} \times \frac{k}{2} \]
22. when \( P \) is at the point (5, 2.5)
   \[ \frac{2k}{\sqrt{3}} \]
23. \( \frac{2k}{\sqrt{3}} \)

Section 3.4, pp. 151–154
1. a. $1.80
   b. $1.07
   c. 5625 L
2. a. 15 terms
   b. 16 terms/h
   c. 20 terms/h
3. a. \( t = 1 \)
   b. 1.5
   c. 
   d. The level will be a maximum.
   e. The level is decreasing.
4. $6000/h when plane is flying at 15 000 m
5. 250 m by 375 m
6. $1100 or $1125
7. 322.50
8. 6 nautical miles/h
9. 20.4 m by 20.4 m by 24.0 m
10. \( r = 4.3 \) cm, \( h = 17.2 \) cm
11. a. $1.5
   b. $12.50, $825
   c. If you increase the price, the number sold will decrease. Profit in situations like this will increase for several price increases and then it will decrease because too many customers stop buying.
12. 12.1 cm by 18.2 cm by 18.2 cm
13. $50
14. $81.25
15. 19 704 units
16. \( P(x) = R(x) - C(x) \)
Marginal Revenue: \( R'(x) \)
Marginal Cost: \( C'(x) \)
Now \( P'(x) = R'(x) - C'(x) \)
The critical point occurs when \( P'(x) = 0 \).
If \( R'(x) = C'(x) \)
then \( P'(x) = R'(x) - C'(x) \)
\[ = 0 \]
Therefore, the instantaneous rate of change in profit is 0 when the marginal revenue equals the marginal cost.
17. \( r = 230 \) cm and \( h \) is about 900 cm
18. 128.4 km/h
19. maximum velocity: \( \frac{4}{27}r^\frac{3}{2}A \), radius: \( \frac{2r_0}{3} \).
Review Exercise, pp. 156–159

1. \( f'(x) = 4x^3 + 4x^{-5} \), \( f''(x) = 12x^2 - 20x^{-6} \)
2. \( \frac{d^2y}{dx^2} = 72t^2 - 42x \)
3. \( v = 2t + (2r - 3)^{-1} \), \( a = 2 - (2r - 3)^{-2} \)
4. \( v(t) = 1 - 5r^2, \) \( a(t) = 10r^3 \)
5. The upward velocity is positive for \( 0 \leq t < 4.5 \) s, zero for \( t = 4.5 \) s, and negative for \( t > 4.5 \) s.

6. a. min: \(-52\), max: 0
   b. min: \(-65\), max: 16
   c. min: 12, max: 20

7. a. 62 m
   b. Yes, 2 m beyond the stop sign
   c. Stop signs are located two or more metres from an intersection. Since the car only went 2 m beyond the stop sign, it is unlikely the car would hit another vehicle travelling perpendicular.

8. min is 2, max is 2 + 3\( \sqrt{3} \)

9. 250

10. a. i. $2200
       ii. $5.50
       iii. $3.00; $3.00
   b. i. $24 640
       ii. $61.60
       iii. $43.20; $43.21
   c. i. $5020
       ii. $12.55
       iii. $0.03: $0.03
   d. i. $27 057
       ii. $6.76
       iii. $4.99; $4.99

11. 2000

12. a. moving away from its starting point

b. moving away from the origin and towards its starting position

13. a. \( t = \frac{2}{3} \)
   b. Yes, since \( a > 0 \) for all \( t > 0 \), the particle is accelerating

14. 27.14 cm by 27.14 cm for the base and height 13.57 cm

15. length 190 m, width approximately 63 m

16. 31.6 m by 11.6 m by 4.2 m

17. radius 4.3 cm, height 8.6 cm

18. Run the pipe 7.2 km along the river shore and then cross diagonally to the refinery.

19. 10:35 p.m.
20. $204 or $206

21. The pipeline meets the shore at a point \( C \), 5.7 km from point \( A \), directly across from \( P \).

22. 11.35 cm by 17.02 cm

23. The two identical brick sides should have length 25 m; the fenced side and the corresponding brick side should have length 40 m.

24. 2:23 p.m.
25. 3.2 km from point \( C \)

26. a. absolute maximum: \( f(7) = 41 \), absolute minimum: \( f(1) = 5 \)
   b. absolute maximum: \( f(3) = 36 \), absolute minimum: \( f(-3) = -18 \)
   c. absolute maximum: \( f(5) = 67 \), absolute minimum: \( f(-5) = -63 \)
   d. absolute maximum: \( f(4) = 2752 \), absolute minimum: \( f(-2) = -56 \)

27. a. 62.9 m
   b. 4.7 s
   c. 3.6 m/s²

28. a. \( f''(2) = 60 \)
   b. \( f''(-1) = 26 \)
   c. \( f''(0) = 192 \)

29. a. position: 1, velocity: \( \frac{1}{6} \), acceleration: \( -\frac{1}{18} \), speed: \( \frac{1}{6} \)
   b. position: \( \frac{8}{3} \), velocity: \( \frac{4}{9} \), acceleration: \( \frac{10}{27} \), speed: \( \frac{4}{9} \)

30. a. \( v(t) = \frac{2}{3}(t^2 + t)^{-\frac{1}{2}}(2t + 1) \), \( a(t) = \frac{2}{9}(t^2 + t)^{-\frac{3}{2}}(2t^2 + 2t - 1) \)
   b. 1.931 m/s
   c. 2.36 m/s
   d. undefined
   e. 0.141 m/s²

Chapter 3 Test, p. 160

1. a. \( y'' = 14 \)
   b. \( f''(x) = -180x^3 - 24x \)
   c. \( y'' = 60x^{-5} + 60x \)
   d. \( f''(x) = 96(4x - 8) \)
2. a. \( v(3) = -57, \) \( a(3) = -44 \)
   b. \( v(2) = 6, \) \( a(2) = -24 \)
3. a. \( v(t) = 2t - 3, \) \( a(t) = 2 \)
   b. \(-0.25 \) m
   c. \( 1 \) m/s
   d. between \( t = 0 \) s and \( t = 1.5 \) s
   e. \( 2 \) m/s
4. a. min: \(-63, \) max: 67
   b. min: 6, max: 10
5. a. 2.1 s
   b. about 22.9 m
6. 250 m by 166.7 m
7. 162 mm by 324 mm by 190 mm
8. $850/month