Answers

Chapter 1

Review of Prerequisite Skills, pp. 2–3

1. **a.** -3 **c.** 4 **e.** -4.1
b. -2 **d.** -4 **f.** $-\frac{1}{2}$

2. **a.** $y = 4x - 2$
b. $y = -2x + 5$
c. $y = \frac{6}{5}(x + 1) + 6$
d. $x + y - 2 = 0$
e. $x = -3$
f. $y = 5$

3. **a.** -1 **c.** -9
b. 0 **d.** 144

4. **a.** $-\frac{5}{52}$ **c.** 0
b. $-\frac{3}{13}$ **d.** $\frac{5}{52}$

5. **a.** 6 **c.** 9
b. $\sqrt{3}$ **d.** $\sqrt{6}$

6. **a.** $-\frac{1}{2}$ **c.** 5 **e.** 10^6
b. -1 **d.** 1

7. **a.** $x^2 - 4x - 12$
b. $15 + 17x - 4x^2$
c. $-x^2 - 7x$
d. $-x^2 + x + 7$
e. $a^3 + 6a^2 + 12a + 8$
f. $729a^3 - 1215a^2 + 675a - 125$

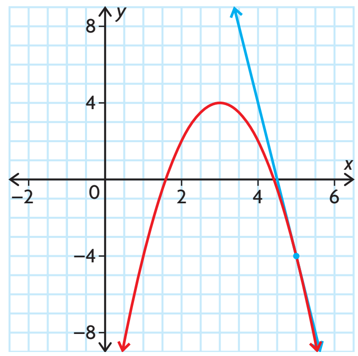
8. **a.** $x(x + 1)(x - 1)$
b. $(x + 3)(x - 2)$
c. $(2x - 3)(x - 2)$
d. $x(x + 1)(x + 1)$
e. $(3x - 4)(9x^2 + 12x + 16)$
f. $(x - 1)(2x - 3)(x + 2)$

9. **a.** $\{x \in \mathbf{R} \mid x \geq -5\}$
b. $\{x \in \mathbf{R}\}$
c. $\{x \in \mathbf{R} \mid x \neq 1\}$
d. $\{x \in \mathbf{R} \mid x \neq 0\}$
e. $\left\{x \in \mathbf{R} \mid x \neq -\frac{1}{2}, 3\right\}$
f. $\{x \in \mathbf{R} \mid x \neq -5, -2, 1\}$

10. **a.** 20.1 m/s **b.** 10.3 m/s

11. **a.** -20 L/min
b. about -13.33 L/min
c. The volume of water in the hot tub is always decreasing during that time period, a negative change.

12. **a.** **b.**



$m = -8$
c. -8

Section 1.1, p. 9

1. **a.** $2\sqrt{3} + 4$ **d.** $3\sqrt{3} - \sqrt{2}$
b. $\sqrt{3} - \sqrt{2}$ **e.** $\sqrt{2} + \sqrt{5}$
c. $-2\sqrt{3} + \sqrt{2}$ **f.** $-\sqrt{5} - 2\sqrt{2}$

2. **a.** $\frac{\sqrt{6} + \sqrt{10}}{2}$ **c.** $\frac{4 + \sqrt{6}}{2}$
b. $\sqrt{6} - 3$ **d.** $\frac{3\sqrt{10} - 2}{4}$

3. **a.** $\sqrt{5} + \sqrt{2}$ **d.** $4 - 2\sqrt{5}$
b. $10 - 3\sqrt{10}$ **e.** $\frac{11\sqrt{6} - 16}{47}$
c. $5 - 2\sqrt{6}$ **f.** $\frac{35 - 12\sqrt{6}}{19}$

4. **a.** $\frac{1}{\sqrt{5} + 1}$
b. $\frac{-7}{2 + 3\sqrt{2}}$
c. $\frac{1}{12 - 5\sqrt{5}}$

5. **a.** $8\sqrt{10} + 24$
b. $8\sqrt{10} + 24$
c. The expressions are equivalent. The radicals in the denominator of part a. have been simplified in part b.

6. **a.** $\sqrt{6} + 2$
b. $\frac{9\sqrt{2} + 2\sqrt{3}}{25}$
c. $2\sqrt{2} + \sqrt{6}$
d. $\frac{12 + 5\sqrt{6}}{2}$
e. $-\frac{12\sqrt{15} + 15\sqrt{10}}{2}$
f. $5 + 2\sqrt{6}$

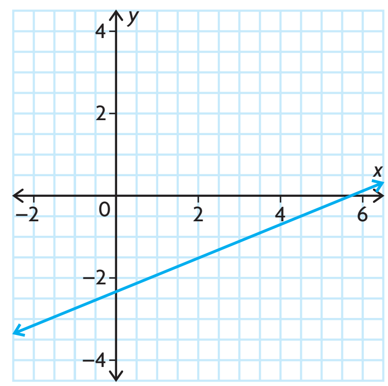
7. **a.** $\frac{1}{\sqrt{a} + 2}$
b. $\frac{1}{\sqrt{x + 4} + 2}$
c. $\frac{1}{\sqrt{x + h} + \sqrt{x}}$

Section 1.2, pp. 18–21

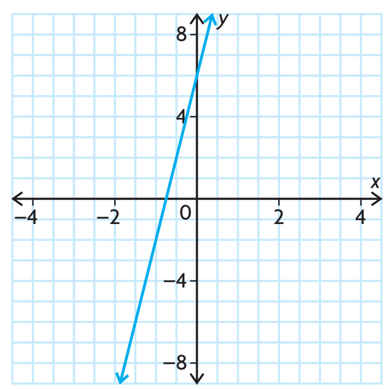
1. **a.** 3 **b.** $-\frac{5}{3}$ **c.** $-\frac{1}{3}$

2. **a.** $-\frac{1}{3}$ **b.** $-\frac{7}{13}$

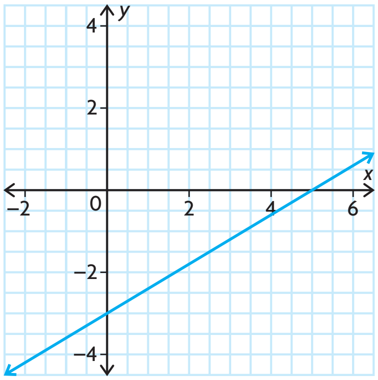
3. **a.** $7x - 17y - 40 = 0$



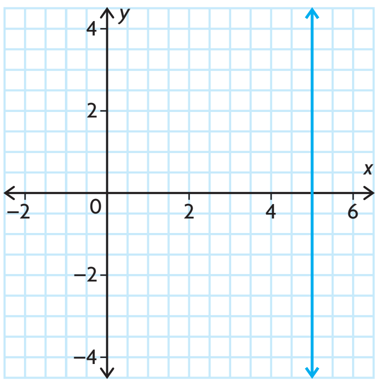
b. $y = 8x + 6$



c. $3x - 5y - 15 = 0$



d. $x = 5$



4. **a.** $75 + 15h + h^2$

b. $108 + 54h + 12h^2 + h^3$

c. $-\dfrac{1}{1 + h}$

d. $6 + 3h$

e. $\dfrac{-3}{4(4 + h)}$

f. $\dfrac{1}{4 + 2h}$

5. **a.** $\dfrac{1}{\sqrt{16 + h} + 4}$

b. $\dfrac{h + 5}{\sqrt{h^2 + 5h + 4} + 2}$

c. $\dfrac{1}{\sqrt{5 + h} + \sqrt{5}}$

6. **a.** $6 + 3h$

b. $3 + 3h + h^2$

c. $\dfrac{1}{\sqrt{9 + h} + 3}$

7. **a.**

P	*Q*	Slope of Line *PQ*
(2, 8)	(3, 27)	19
(2, 8)	(2.5, 15.625)	15.25
(2, 8)	(2.1, 9.261)	12.61
(2, 8)	(2.01, 8.120 601)	12.0601
(2, 8)	(1, 1)	7
(2, 8)	(1.5, 3.375)	9.25
(2, 8)	(1.9, 6.859)	11.41
(2, 8)	(1.99, 7.880 599)	11.9401

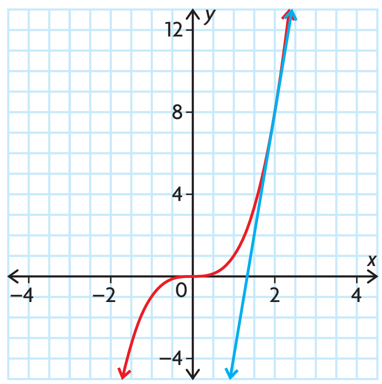
b. 12

c. $12 + 6h + h^2$

d. 12

e. They are the same.

f.



8. **a.** -12 **b.** 5 **c.** 12

9. **a.** $\dfrac{1}{2}$ **b.** $\dfrac{1}{4}$ **c.** $\dfrac{5}{6}$

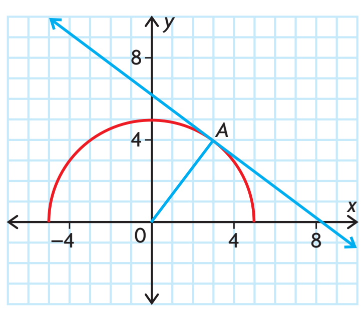
10. **a.** -2 **b.** $-\dfrac{1}{2}$ **c.** $-\dfrac{1}{25}$

11. **a.** 1 **d.** $\dfrac{1}{6}$

b. -1 **e.** $-\dfrac{3}{4}$

c. 9 **f.** $-\dfrac{1}{6}$

12.



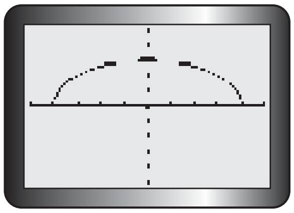
$y = \sqrt{25 - x^2} \rightarrow$ Semi-circle centre (0, 0), rad 5, $y \geq 0$

OA is a radius. The slope of *OA* is $\frac{4}{3}$.

The slope of tangent is $-\frac{3}{4}$.

13. Take values of x close to the point, then determine $\frac{\Delta y}{\Delta x}$.

14.



Since the tangent is horizontal, the slope is 0.

15. $3x - y - 8 = 0$

16. $3x + y - 8 = 0$

17. **a.** $(3, -2)$

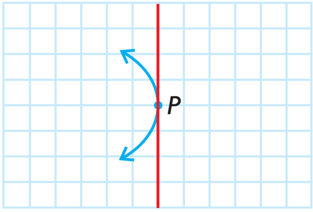
b. $(5, 6)$

c. $y = 4x - 14$

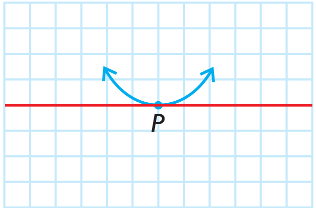
d. $y = 2x - 8$

e. $y = 6x - 24$

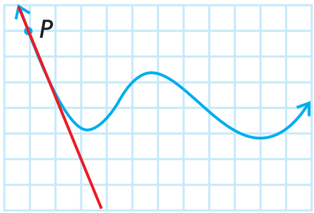
18. **a.** undefined



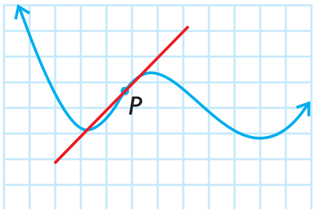
b. 0



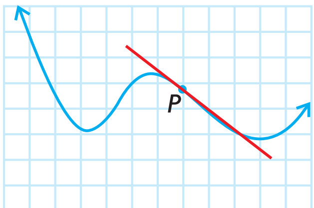
c. about -2.5



d. about 1



e. about $-\frac{7}{8}$



f. no tangent at point P

19. $-\frac{5}{4}$

20. 500 papers/year

21. $(2, 4)$

22. $\left(-2, \frac{28}{3}\right), \left(-1, \frac{26}{3}\right), \left(1, -\frac{26}{3}\right), \left(2, -\frac{28}{3}\right)$

23. $y = x^2$ and $y = \frac{1}{2} - x^2$

$$x^2 = \frac{1}{2} - x^2$$

$$x^2 = \frac{1}{4}$$

$$x = \frac{1}{2} \text{ or } x = -\frac{1}{2}$$

The points of intersection are $P\left(\frac{1}{2}, \frac{1}{4}\right)$ and $Q\left(-\frac{1}{2}, \frac{1}{4}\right)$.

Tangent to $y = x$:

$$m = \lim_{h \to 0} \frac{(a + h)^2 - a^2}{h}$$

$$= \lim_{h \to 0} \frac{2ah + h^2}{h}$$

$$= 2a$$

The slope of the tangent at $a = \frac{1}{2}$ is $1 = m_p$ and at $a = -\frac{1}{2}$ is $-1 = m_q$.

Tangents to $y = \frac{1}{2} - x^2$:

$$m = \lim_{h \to 0} \frac{\left[\frac{1}{2} - (a + h)^2\right] - \left[\frac{1}{2} - a^2\right]}{h}$$

$$= \lim_{h \to 0} \frac{-2ah - h^2}{h}$$

$$= -2a$$

The slope of the tangents at $a = \frac{1}{2}$ is $-1 = M_P$ and at $a = -\frac{1}{2}$ is $1 = M_q$; $m_p M_P = -1$ and $m_q M_q = -1$. Therefore, the tangents are perpendicular at the points of intersection.

24. $y = -11x + 24$

25. **a.** $8a + 5$
b. $(0, -2)$
c. $(-5, 73)$

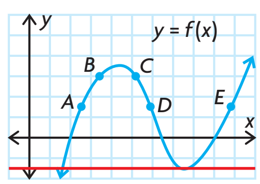
Section 1.3, pp. 29–31

1. 0 s or 4 s

2. **a.** Slope of the secant between the points $(2, s(2))$ and $(9, s(9))$
b. Slope of the tangent at the point $(6, s(6))$

3. Slope of the tangent to the function with equation $y = \sqrt{x}$ at the point $(4, 2)$

4. **a.** A and B
b. greater; the secant line through these two points is steeper than the tangent line at B.
c.



5. Speed is represented only by a number, not a direction.

6. Yes, velocity needs to be described by a number and a direction. Only the speed of the school bus was given, not the direction, so it is not correct to use the word "velocity."

7. **a.** first second = − 5 m/s, third second = − 25 m/s, eighth second = − 75 m/s
b. −55 m/s
c. −20 m/s

8. **a.** **i.** 72 km/h
ii. 64.8 km/h
iii. 64.08 km/h
b. 64 km/h
c. 64 km/h

9. **a.** 15 terms
b. 16 terms/h

10. **a.** $-\frac{1}{3}$ mg/h
b. Amount of medicine in 1 mL of blood being dissipated throughout the system

11. $\frac{1}{50}$ s/m

12. $-\frac{12}{5}$ °C/km

13. 2 s; 0 m/s

14. **a.** $4800
b. $80 per ball
c. $0 < x < 80$

15. **a.** 6
b. −1
c. $\frac{1}{10}$

16. $1 162 250/year

17. **a.** 75 m
b. 30 m/s
c. 60 m/s
d. 14 s

18. The coordinates of the point are $\left(a, \frac{1}{a}\right)$.

The slope of the tangent is $-\frac{1}{a^2}$. The equation of the tangent is $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$, or $y = -\frac{1}{a^2}x + \frac{2}{a}$. The intercepts are $\left(0, \frac{2}{a}\right)$ and $(-2a, 0)$. The tangent line and the axes form a right triangle with legs of length $\frac{2}{a}$ and $2a$. The area of the triangle is $\frac{1}{2}\left(\frac{2}{a}\right)(2a) = 2$.

19.
$$C(x) = F + V(x)$$
$$C(x + h) = F + V(x + h)$$

Rate of change of cost is

$$\lim_{h \to 0} \frac{C(x + h) - C(x)}{h}$$

$$= \lim_{h \to 0} \frac{V(x + h) - V(x)}{h},$$

which is independent of F − (fixed costs)

20. 200π m²/m

21. Cube of dimensions x by x by x has volume $V = x^3$. Surface area is $6x^2$. $V'(x) = 3x^2 = \frac{1}{2}$ surface area.

22. **a.** 80π cm²/unit of time
b. -100π cm³/unit of time

Mid-Chapter Review, pp. 32–33

1. **a.** 3 **c.** 61
b. 37 **d.** 5

2. **a.** $\frac{6\sqrt{3} + \sqrt{6}}{3}$
b. $\frac{6 + 4\sqrt{3}}{3}$
c. $-\frac{5(\sqrt{7} + 4)}{9}$
d. $-2(3 + 2\sqrt{3})$

e. $\dfrac{10\sqrt{3} - 15}{2}$

f. $-\dfrac{3\sqrt{2}(2\sqrt{3} + 5)}{13}$

3. a. $\dfrac{2}{5\sqrt{2}}$

b. $\dfrac{3}{\sqrt{3}(6 + \sqrt{2})}$

c. $-\dfrac{9}{5(\sqrt{7} + 4)}$

d. $-\dfrac{13}{3\sqrt{2}(2\sqrt{3} + 5)}$

e. $-\dfrac{1}{(\sqrt{3} + \sqrt{7})}$

f. $\dfrac{1}{(2\sqrt{3} - \sqrt{7})}$

4. a. $\frac{2}{3}x + y - 6 = 0$

b. $x - y + 5 = 0$

c. $4x - y - 2 = 0$

d. $x - 5y - 9 = 0$

5. -2

6. a.

P	Q	Slope of Line PQ
(−1, 1)	(−2, 6)	−5
(−1, 1)	(−1.5, 3.25)	−4.5
(−1, 1)	(−1.1, 1.41)	−4.1
(−1, 1)	(−1.01, 1.0401)	−4.01
(−1, 1)	(−1.001, 1.004 001)	−4.001

P	Q	Slope of Line PQ
(−1, 1)	(0, −2)	−3
(−1, 1)	(−0.5, −0.75)	−3.5
(−1, 1)	(−0.9, 0.61)	−3.9
(−1, 1)	(−0.99, 0.960 1)	−3.99
(−1, 1)	(−0.999, 0.996 001)	−3.999

b. -4

c. $h - 4$

d. -4

e. The answers are equal.

7. a. -3 c. $-\frac{1}{4}$

b. -9 d. $\frac{1}{6}$

8. a. i. 36 km/h
ii. 30.6 km/h
iii. 30.06 km/h

b. velocity of car appears to approach 30 km/h

c. $(6h + 30)$ km/h

d. 30 km/h

9. a. -4

b. -12

10. a. -2000 L/min

b. -1000 L/min

11. a. $-9x + y + 19 = 0$

b. $8x + y + 15 = 0$

c. $4x + y + 8 = 0$

d. $-2x + y + 2 = 0$

12. a. $-3x + 4y - 25 = 0$

b. $3x + 4y + 5 = 0$

Section 1.4, pp. 37–39

1. a. $\frac{72}{99}$ b. π

2. Evaluate the function for values of the independent variable that get progressively closer to the given value of the independent variable.

3. a. A right-sided limit is the value that a function gets close to as the values of the independent variable decrease and get close to a given value.

b. A left-sided limit is the value that a function gets close to as the values of the independent variable increase and get close to a given value.

c. A (two-sided) limit is the value that a function gets close to as the values of the independent variable get close to a given value, regardless of whether the values increase or decrease toward the given value.

4. a. -5 d. -8

b. 10 e. 4

c. 100 f. 8

5. 1

6. a. 0 c. -1

b. 2 d. 2

7. a. 2

b. 1

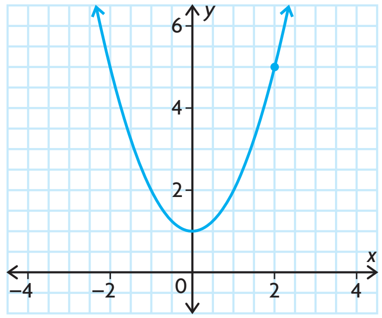
c. does not exist

8. a. 8

b. 2

c. 2

9. 5



10. a. 0 d. $-\frac{1}{2}$

b. 0 e. $\frac{1}{5}$

c. 5

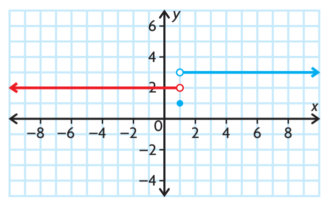
f. does not exist; substitution causes division by zero, and there is no way to remove the factor from the denominator.

11. a. does not exist c. 2

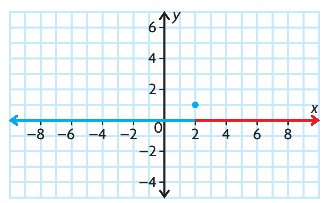
b. 2 d. does not exist

12. Answers may vary. For example:

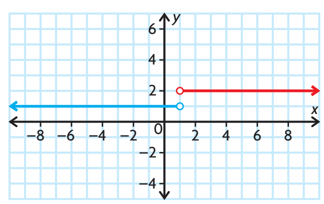
a.



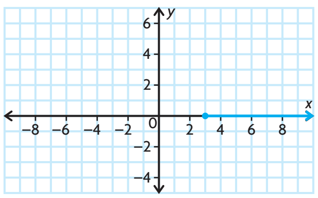
b.



c.



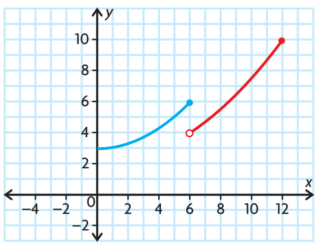
d.



13. $m = -3; b = 1$

14. $a = 3, b = 2, c = 0$

15. **a.**



b. 6; 4

c. 2000

d. about 8.49 years

Section 1.5, pp. 45–47

1. $\lim_{x\to 2}(3 + x)$ and $\lim_{x\to 2}(x + 3)$ have the same value, but $\lim_{x\to 2} 3 + x$ does not. Since there are no brackets around the expression, the limit only applies to 3, and there is no value for the last term, x.

2. Factor the numerator and denominator. Cancel any common factors. Substitute the given value of x.

3. Yes, if the two one-sided limits have the same value, then the value of the limit is equal to the value of the one-sided limits. If the one-sided limits do not have the same value, then the limit does not exist.

4. **a.** 1 **d.** $5\pi^3$
b. 1 **e.** 2
c. $\frac{100}{9}$ **f.** $\sqrt{3}$

5. **a.** 2
b. $\sqrt{2}$

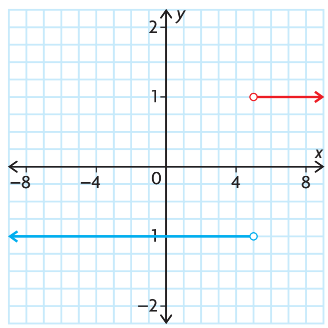
6. Since substituting $t = 1$ does not make the denominator 0, direct substitution works. $\frac{1 - 1 - 5}{6 - 1} = \frac{-5}{5}$
$= -1$

7. **a.** 4 **d.** $-\frac{1}{4}$
b. 1 **e.** $\frac{1}{4}$
c. 27 **f.** $-\frac{1}{\sqrt{7}}$

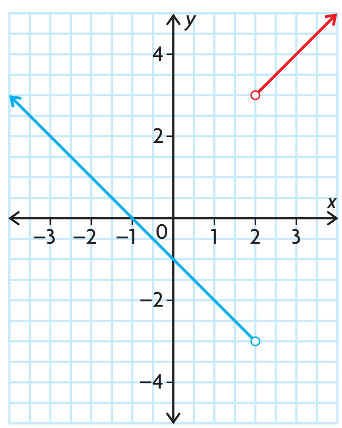
8. **a.** $\frac{1}{12}$ **d.** $\frac{1}{2}$
b. -27 **e.** $\frac{1}{12}$
c. $\frac{1}{6}$ **f.** $\frac{1}{12}$

9. **a.** 0 **d.** $\frac{1}{2}$
b. 0 **e.** $2x$
c. -1 **f.** $\frac{1}{32}$

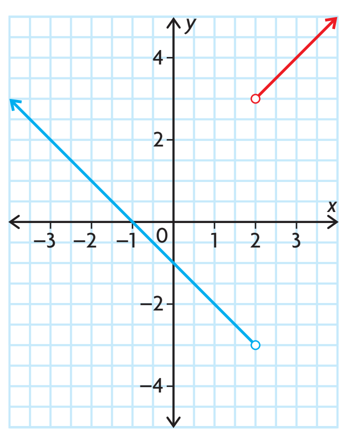
10. **a.** does not exist



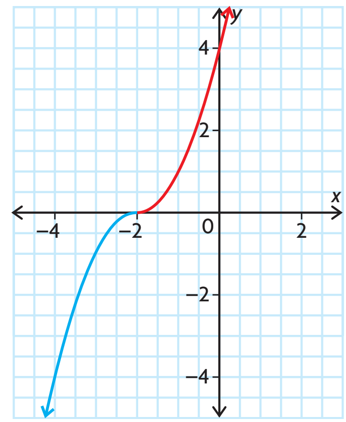
b. does not exist



c. does not exist



d. exists



11. **a.**

ΔT	T	V	ΔV
	−40	19.1482	
20			1.6426
	−20	20.7908	
20			1.6426
	0	22.4334	
20			1.6426
	20	24.0760	
20			1.6426
	40	25.7186	
20			1.6426
	60	27.3612	
20			1.6426
	80	29.0038	

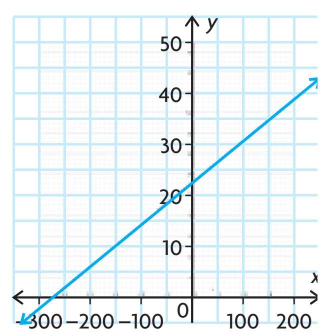
ΔV is constant; therefore, T and V form a linear relationship.

b. $V = 0.082\,13T + 22.4334$

c. $T = \frac{V - 22.4334}{0.082\,13}$

d. $\lim_{v\to 0^+} T = \frac{0 - 22.4334}{0.082\,13}$
$= -273.145$

e.



12. $\lim_{x\to 5}\frac{x^2 - 4}{f(x)}$
$= \frac{\lim_{x\to 5}(x^2 - 4)}{\lim_{x\to 5} f(x)}$
$= \frac{21}{3}$
$= 7$

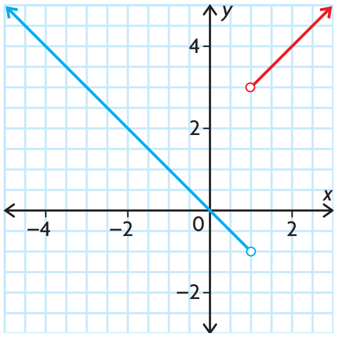
13. **a.** 27 **b.** -1 **c.** 1

14. **a.** 0 **b.** 0

15. **a.** 0 **b.** $\frac{1}{2}$

16. -2

17. does not exist

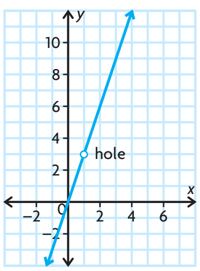


Section 1.6, pp. 51–53

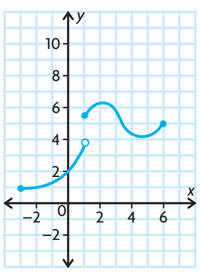
1. Anywhere that you can see breaks or jumps is a place where the function is not continuous.

2. On a given domain, you can trace the graph of the function without lifting your pencil.

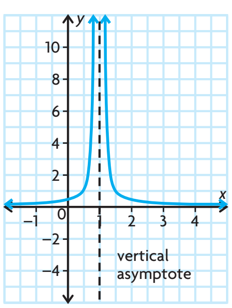
3. point discontinuity



jump discontinuity



infinite discontinuity

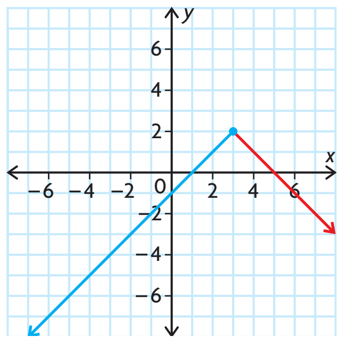


4. **a.** $x = 3$
b. $x = 0$
c. $x = 0$
d. $x = 3$ and $x = -3$
e. $x = -3$ and $x = 2$
f. $x = 3$

5. **a.** continuous for all real numbers
b. continuous for all real numbers
c. continuous for all real numbers, except 0 and 5
d. continuous for all real numbers greater than or equal to -2
e. continuous for all real numbers
f. continuous for all real numbers

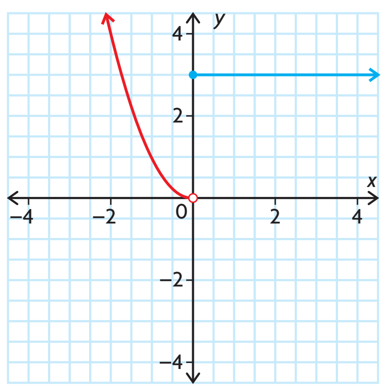
6. $g(x)$ is a linear function (a polynomial), and so is continuous everywhere, including $x = 2$.

7.



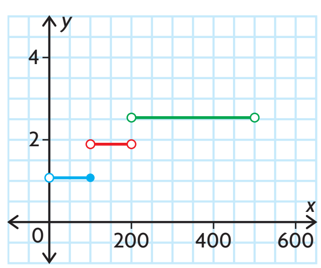
Yes, the function is continuous everywhere.

8.



The function is discontinuous at $x = 0$.

9.



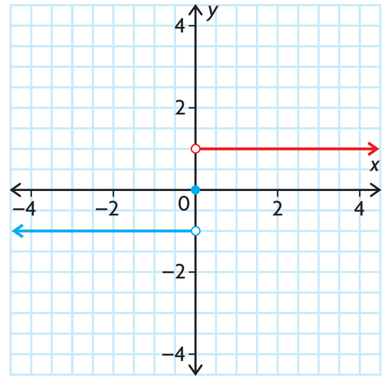
Discontinuities at 0, 100, 200, and 500

10. no

11. Discontinuous at $x = 2$

12. $k = 16$

13. **a.**



b. **i.** -1
ii. 1
iii. does not exist

c. f is not continuous since $\lim_{x\to 0} f(x)$ does not exist.

14. **a.** 2
b. 4
c. $\lim_{x\to 3^-} f(x) = 4 = \lim_{x\to 3^+} f(x)$
Thus, $\lim_{x\to 3} f(x) = 4$. But, $f(3) = 2$.
Hence, f is not continuous at $x = 3$ and also not continuous on $-3 < x < 8$.

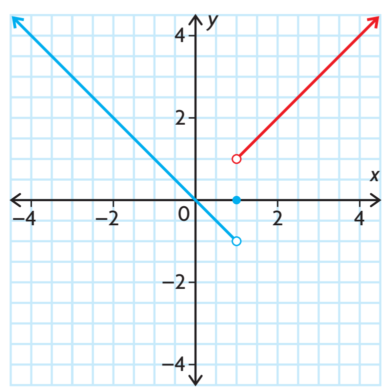
15. (1) $A = B - 3$
(2) $4B - A \neq 6$ (if $B > 1$, then $A > -2$; if $B < 1$, then $A < -2$)

16. $a = -1, b = 6$

17. **a.** $\left.\begin{array}{l}\lim_{x\to 1^-} g(x) = -1 \\ \lim_{x\to 1^+} g(x) = 1\end{array}\right\} \lim_{x\to 1} g(x)$

$\lim_{x\to 1} g(x)$ does not exist.

b.



$g(x)$ is discontinuous at $x = 1$.

Review Exercise, pp. 56–59

1. **a.** -3 **b.** 7
c. $2x - y - 5 = 0$

2. **a.** $\frac{-1}{3}$ **c.** $-\frac{1}{27}$
b. $\frac{1}{2}$ **d.** $-\frac{5}{4}$

3. **a.** 2 **b.** 2

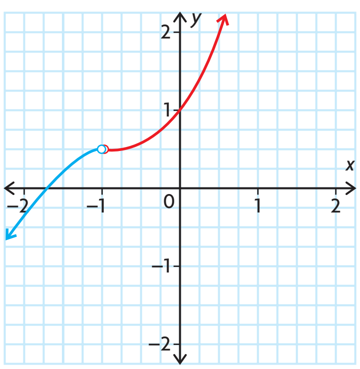
4. **a.** 1st second $= -5$ m/s,
2nd second $= -15$ m/s
b. -40 m/s
c. -60 m/s

5. **a.** 0.0601 g
b. 6.01 g/min
c. 6 g/min

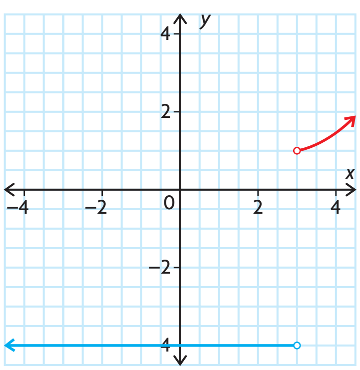
6. **a.** 700 000 t
b. 18×10^4 t per year
c. 15×10^4 t per year
d. 7.5 years

7. **a.** 10
b. 7; 0
c. $t = 3$ and $t = 4$

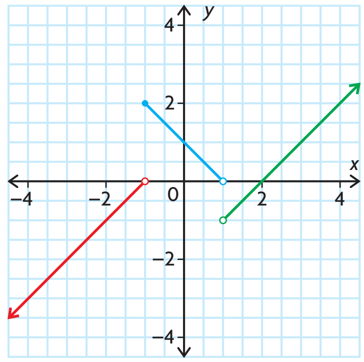
8. **a.** Answers may vary. For example:



b. Answers may vary. For example:



9. **a.**



b. $x = -1$ and $x = 1$
c. They do not exist.

10. not continuous at $x = -4$

11. **a.** $x = 1$ and $x = -2$
b. $\lim_{x\to 1} f(x) = \frac{2}{3}$,
$\lim_{x\to -2} f(x)$ does not exist.

12. **a.** $\lim_{x\to 0} f(x)$ does not exist.
b. $\lim_{x\to 0} g(x) = 0$
c. $\lim_{x\to -3} h(x) = \frac{37}{7}$
$\lim_{x\to -3} h(x)$ does not exist.

13. **a.**

x	1.9	1.99	1.999	2.001	2.01	2.1
$\frac{x-2}{x^2-x-2}$	0.344 83	0.334 45	0.333 44	0.333 22	0.332 23	0.322 58

$\frac{1}{3}$

b.

x	0.9	0.99	0.999	1.001	1.01	1.1
$\frac{x-1}{x^2-1}$	0.526 32	0.502 51	0.500 25	0.499 75	0.497 51	0.476 19

$\frac{1}{2}$

14.

x	−0.1	−0.01	−0.001	0.001	0.01	0.1
$\frac{\sqrt{x+3}-\sqrt{3}}{x}$	0.291 12	0.288 92	0.2887	0.288 65	0.288 43	0.286 31

$\frac{1}{2\sqrt{3}}$; This agrees well with the values in the table.

15. **a.**

x	2.1	2.01	2.001	2.0001
$f(x)$	0.248 46	0.249 84	0.249 98	0.25

$\lim_{x\to 2} f(x) \doteq 0.25$

b. $\lim_{x\to 2} f(x) = 0.25$
c. 0.25

16. **a.** 10 **b.** $\frac{1}{4}$ **c.** $-\frac{1}{16}$

17. **a.** 4 **c.** $\frac{1}{\sqrt{5}}$ **e.** $-\frac{1}{8}$
b. $10a$ **d.** $\frac{1}{3}$ **f.** $-\frac{1}{4}$

18. **a.** The function is not defined for $x < 3$, so there is no left-side limit.
b. Even after dividing out common factors from numerator and denominator, there is a factor of $x - 2$ in the denominator; the graph has a vertical asymptote at $x = 2$.
c. $\lim_{x\to 1^-} f(x) = -5 \neq \lim_{x\to 1^+} f(x) = 2$
d. The function has a vertical asymptote at $x = 2$.
e. $x \to 0^-$ $|x| = -x$

$\lim_{x\to 0^-} \frac{|x|}{x} = -1$

$\lim_{x\to 0^+} \frac{|x|}{x} = 1$

$\lim_{x\to 0^+} \frac{|x|}{x} \neq \lim_{x\to 0^-} \frac{|x|}{x}$

f. $\lim_{x\to -1^+} f(x) = -1$
$\lim_{x\to -1^-} f(x) = 5$
$\lim_{x\to -1^+} f(x) \neq \lim_{x\to -1^-} f(x)$
Therefore, $\lim_{x\to -1} f(x)$ does not exist.

19. **a.** $y = 7$
b. $y = -5x - 5$
c. $y = 18x + 9$
d. $y = -216x + 486$

20. **a.** 700 000
b. 109 000/h

Chapter 1 Test, p. 60

1. $\lim\limits_{x\to1^+} \frac{1}{x-1} = +\infty \neq \lim\limits_{x\to1^-} \frac{1}{x-1} = -\infty$

2. -13

3. **a.** $\lim\limits_{x\to1} f(x)$ does not exist.
b. 1
c. 1
d. $x = 1$ and $x = 2$

4. **a.** 1 km/h
b. 2 km/h

5. $\frac{\sqrt{16+h}-4}{h}$

6. -31

7. **a.** 12 **d.** $-\frac{3}{4}$
b. $\frac{7}{5}$ **e.** $\frac{1}{6}$
c. 4 **f.** $\frac{1}{12}$

8. **a.** $a = 1, b = -\frac{18}{5}$

Chapter 2

Review of Prerequisite Skills, pp. 62–63

1. **a.** a^8 **d.** $\frac{1}{a^2b^7}$
b. $-8a^6$ **e.** $48e^{18}$
c. $2p$ **f.** $-\frac{b}{2a^6}$

2. **a.** $x^{\frac{7}{6}}$ **b.** $4x^4$ **c.** $a^{\frac{1}{3}}$

3. **a.** $-\frac{3}{2}$ **c.** $-\frac{3}{5}$
b. 2 **d.** 1

4. **a.** $x - 6y - 21 = 0$
b. $3x - 2y - 4 = 0$
c. $4x + 3y - 7 = 0$

5. **a.** $2x^2 - 5xy - 3y^2$
b. $x^3 - 5x^2 + 10x - 8$
c. $12x^2 + 36x - 21$
d. $-13x + 42y$
e. $29x^2 - 2xy + 10y^2$
f. $-13x^3 - 12x^2y + 4xy^2$

6. **a.** $\frac{15}{2}x; x \neq 0, -2$
b. $\frac{y-5}{4y^2(y+2)}; y \neq -2, 0, 5$
c. $\frac{8}{9}; h \neq -k$
d. $\frac{2}{(x+y)^2}; x \neq -y, +y$
e. $\frac{11x^2 - 8x + 7}{2x(x-1)}; x \neq 0, 1$
f. $\frac{4x+7}{(x+3)(x-2)}; x \neq -3, 2$

7. **a.** $(2k+3)(2k-3)$
b. $(x-4)(x+8)$
c. $(a+1)(3a-7)$
d. $(x^2+1)(x+1)(x-1)$
e. $(x-y)(x^2+xy+y^2)$
f. $(r+1)(r-1)(r+2)(r-2)$

8. **a.** $(a-b)(a^2+ab+b^2)$
b. $(a-b)(a^4+a^3b+a^2b^2 + ab^3+b^4)$
c. $(a-b)(a^6+a^5b+a^4b^2 + a^3b^3+a^2b^4+ab^5+b^6)$
d. $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + a^2b^{n-3} + ab^{n-2} + b^{n-1})$

9. **a.** -17 **c.** $\frac{53}{8}$
b. 10 **d.** about 7.68

10. **a.** $\frac{3\sqrt{2}}{2}$
b. $\frac{4\sqrt{3}-\sqrt{6}}{3}$
c. $-\frac{30+17\sqrt{2}}{23}$
d. $-\frac{11-4\sqrt{6}}{5}$

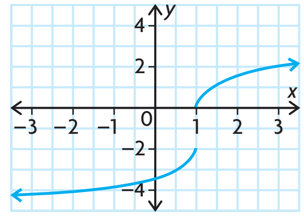
11. **a.** $3h + 10$; expression can be used to determine the slope of the secant line between $(2, 8)$ and $(2 + h, f(2 + h))$
b. For $h = 0.01$: 10.03
c. value represents the slope of the secant line through $(2, 8)$ and $(2.01, 8.1003)$

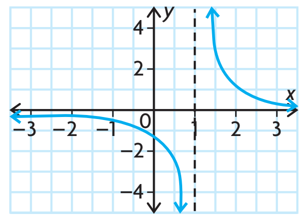
Section 2.1, pp. 73–75

1. **a.** $\{x \in \mathbf{R} \mid x \neq -2\}$
b. $\{x \in \mathbf{R} \mid x \neq 2\}$
c. $\{x \in \mathbf{R}\}$
d. $\{x \in \mathbf{R} \mid x \neq 1\}$
e. $\{x \in \mathbf{R}\}$
f. $\{x \in \mathbf{R} \mid x > 2\}$

2. The derivative of a function represents the slope of the tangent line at a give value of the independent variable or the instantaneous rate of change of the function at a given value of the independent variable.

3. Answers may vary. For example:





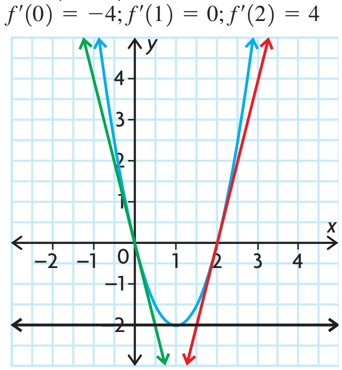
4. **a.** $5a + 5h - 2; 5h$
b. $a^2 + 2ah + h^2 + 3a + 3h - 1; 2ah + h^2 + 3h$
c. $a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1; 3a^2h + 3ah^2 + h^3 - 4h$
d. $a^2 + 2ah + h^2 + a + h - 6; 2ah + h^2 + h$
e. $-7a - 7h + 4; -7h$
f. $4 - 2a - 2h - a^2 - 2ah - h^2; -2h - h^2 - 2ah$

5. **a.** 2 **c.** $\frac{1}{2}$
b. 9 **d.** -5

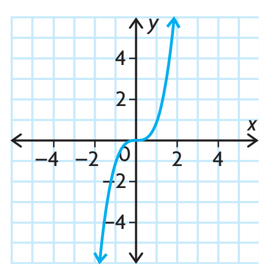
6. **a.** -5 **c.** $18x^2 - 7$
b. $4x + 4$ **d.** $\frac{3}{2\sqrt{3x+2}}$

7. **a.** -7
b. $-\frac{2}{(x-1)^2}$ **c.** $6x$

8. $f'(0) = -4; f'(1) = 0; f'(2) = 4$

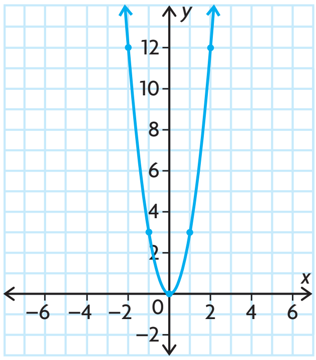


9. **a.**



b. $f'(-2) = 12; f'(-1) = 3;$
$f'(0) = 0; f'(1) = 3; f'(2) = 12$

c.



d. graph of $f(x)$ is cubic; graph of $f'(x)$ seems to be a parabola

10. $s'(0) = 8$ m/s; $s'(4) = 0$ m/s; $s'(6) = -4$ m/s

11. $x - 6y + 10 = 0$

12. **a.** 0 **c.** m
b. 1 **d.** $2ax + b$

13. Since $3x^2$ is nonnegative for all x, the original function never has a negative slope.

14. **a.** -1.6 m/s
b. $h'(2)$ measures the rate of change in the height of the ball with respect to time when $t = 2$.

15. **a.** e. **b.** f. **c.** d.

16. $\lim_{h \to 0^-} \frac{f(0 + h) - f(0)}{h}$

$= \lim_{h \to 0^-} \frac{-(0 + h)^2 - (-0^2)}{h}$

$= \lim_{h \to 0^-} \frac{-h^2}{h}$

$= \lim_{h \to 0^-} (-h)$

$= 0$

$\lim_{h \to 0^+} \frac{f(0 + h) - f(0)}{h}$

$= \lim_{h \to 0^+} \frac{(0 + h)^2 - (0^2)}{h}$

$= \lim_{h \to 0^+} \frac{h^2}{h}$

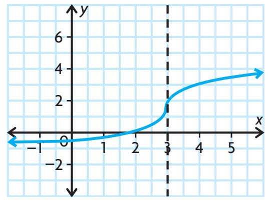
$= \lim_{h \to 0^+} (h)$

$= 0$

Since the limits are equal for both sides, the derivative exists and $f'(0) = 0$.

17. 3

18. Answers may vary. For example:



19. $(3, -8)$

20. $2x + y + 1 = 0$ and $6x - y - 9 = 0$

Section 2.2, pp. 82–84

1. Answers may vary. For example:
constant function rule: $\frac{d}{dx}(5) = 0$
power rule: $\frac{d}{dx}(x^3) = 3x^2$
constant multiple rule: $\frac{d}{dx}(4x^3) = 12x^2$
sum rule: $\frac{d}{dx}(x^2 + x) = 2x + 1$
difference rule: $\frac{d}{dx}(x^3 - x^2 + 3x) = 3x^2 - 2x + 3$

2. **a.** 4 **d.** $\frac{1}{3\sqrt[3]{x^2}}$
b. $3x^2 - 2x$ **e.** $\frac{x^3}{4}$
c. $-2x + 5$ **f.** $-3x^{-4}$

3. **a.** $4x + 11$ **d.** $x^4 + x^2 - x$
b. $6x^2 + 10x - 4$ **e.** $40x^7$
c. $4t^3 - 6t^2$ **f.** $2t^3 - \frac{3}{2}$

4. **a.** $5x^{\frac{2}{3}}$
b. $-2x^{-\frac{3}{2}} + 6x^{-2}$
c. $\frac{-18}{x^4} - \frac{4}{x^3}$
d. $-18x^{-3} + \frac{3}{2}x^{-\frac{1}{2}}$
e. $\frac{1}{2}(x^{-\frac{1}{2}}) + 9x^{\frac{1}{2}}$
f. $-x^{-2} - \frac{1}{2}x^{-\frac{3}{2}}$

5. **a.** $-4t + 7$ **b.** $5 - t^2$ **c.** $2t - 6$

6. **a.** 47.75 **b.** $\frac{11}{24}$

7. **a.** 12 **c.** $-\frac{1}{2}$
b. 5 **d.** 12

8. **a.** 9 **c.** 4
b. $\frac{1}{2}$ **d.** -7

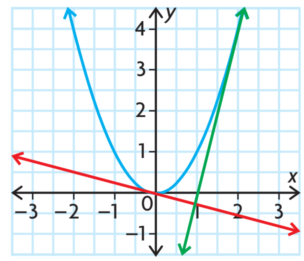
9. **a.** $6x - y - 4 = 0$
b. $18x - y + 25 = 0$
c. $9x - 2y - 9 = 0$
d. $x + y - 3 = 0$
e. $7x - 2y - 28 = 0$
f. $5x - 6y - 11 = 0$

10. A normal to the graph of a function at a point is a line that is perpendicular to the tangent at the given point;
$x + 18y - 125 = 0$

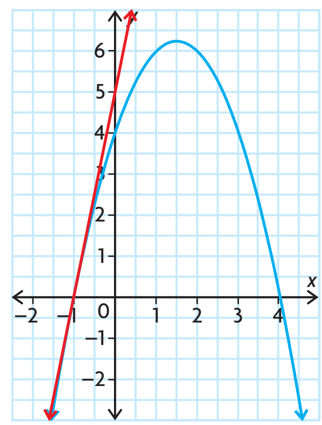
11. 8

12. no

13. $y = x^2, \frac{dy}{dx} = 2x$
The slope of the tangent at $A(2, 4)$ is 4 and at $B\left(-\frac{1}{8}, \frac{1}{64}\right)$ is $-\frac{1}{4}$.
Since the product of the slopes is -1, the tangents at $A(2, 4)$ and $B\left(-\frac{1}{8}, \frac{1}{64}\right)$ will be perpendicular.



14. $(-1, 0)$



15. $(2, 10)$ and $(-2, -6)$

16. $y = \frac{1}{5}x^5 - 10x$, slope is 6
$\frac{dy}{dx} = x^4 - 10 = 6$
$x^4 = 16$
$x^2 = 4$ or $x^2 = -4$
$x = \pm 2$ non-real
Tangents with slope 6 are at the points $\left(2, -\frac{68}{5}\right)$ and $\left(-2, \frac{68}{5}\right)$.

17. **a.** $y - 3 = 0; 16x - y - 29 = 0$
b. $20x - y - 47 = 0; 4x + y - 1 = 0$

18. 7

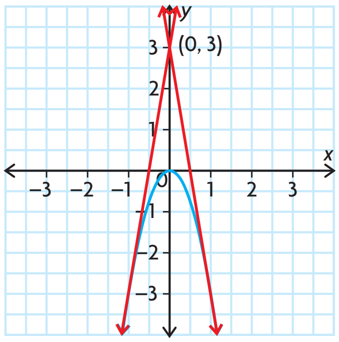
19. **a.** 49.9 km
b. 0.12 km/m

20. **a.** 34.3 m/s
b. 39.2 m/s
c. 54.2 m/s

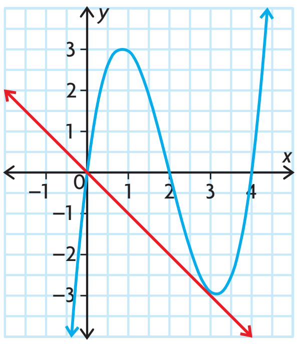
21. 0.29 min and 1.71 min

22. -20 m/s

23. $(1, -3)$ and $(-1, -3)$



24. $B(0, 0)$



25. **a.** **i.** $\left(\frac{1}{5}, \frac{1}{5}\right)$

ii. $\left(-\frac{1}{4}, -\frac{13}{4}\right)$

iii. $\left(\frac{1}{3}, \frac{103}{27}\right)$ and $(5, -47)$

b. At these points, the slopes of the tangents are zero, meaning that the rate of change of the value of the function with respect to the domain is zero. These points are also local maximum and minimum points.

26. $\sqrt{x} + \sqrt{y} = 1$

$P(a, b)$ is on the curve; therefore, $a \geq 0, b \geq 0$.

$\sqrt{y} = 1 - \sqrt{x}$

$y = 1 - 2\sqrt{x} + x$

$\frac{dy}{dx} = -\frac{1}{2}\left(2x^{-\frac{1}{2}} + 1\right)$

At $x = a$. Slope is

$-\frac{1}{\sqrt{a}} + 1 = \frac{-1 + \sqrt{a}}{\sqrt{a}}$.

But, $\sqrt{a} + \sqrt{b} = 1$

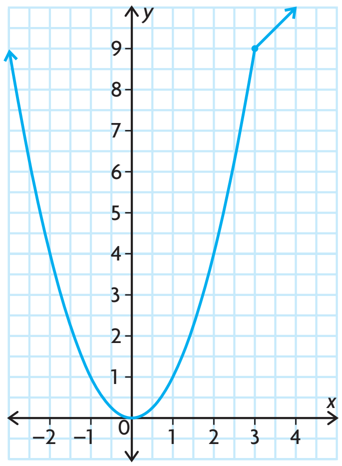
$-\sqrt{b} = \sqrt{a} - 1$

Therefore, slope is $-\frac{\sqrt{b}}{\sqrt{a}} = -\sqrt{\frac{b}{a}}$.

27. The x-intercept is $1 - \frac{1}{n}$, as $n \to \infty$, $\frac{1}{n} \to 0$, and the x-intercept approaches 1. As $n \to \infty$, the slope of the tangent at $(1, 1)$ increase without bound, and the tangent approaches a vertical line having equation $x - 1 = 0$.

28. **a.** $f'(x) = \begin{cases} 2x, \text{ if } x < 3 \\ 1, \text{ if } x \geq 3 \end{cases}$

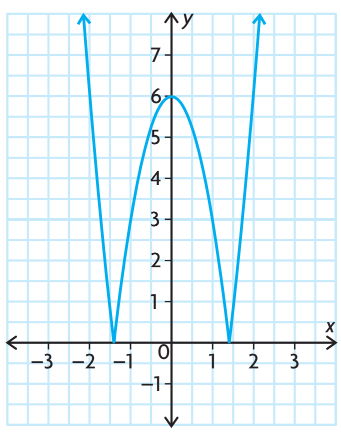
$f'(3)$ does not exist.



b.

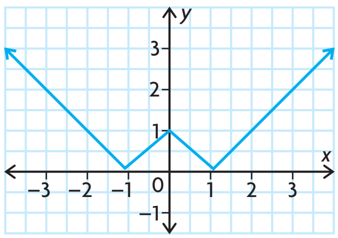
$f'(x) = \begin{cases} 6x, \text{ if } x < -\sqrt{2} \text{ or } x > \sqrt{2} \\ -6x, \text{ if } -\sqrt{2} \leq x \leq \sqrt{2} \end{cases}$

$f'\left(\sqrt{2}\right)$ and $f'\left(-\sqrt{2}\right)$ do not exist.



c. $f'(x) = \begin{cases} 1, \text{ if } x > 1 \\ -1, \text{ if } 0 < x < 1 \\ 1, \text{ if } -1 < x < 0 \\ -1, \text{ if } x < -1 \end{cases}$

$f'(0)$, $f'(-1)$, and $f'(1)$ do not exist.



Section 2.3, pp. 90–91

1. **a.** $2x - 4$
b. $6x^2 - 2x$
c. $12x - 17$
d. $45x^8 - 80x^7 + 2x - 2$
e. $-8t^3 + 2t$
f. $\frac{6}{(x + 3)^2}$

2. **a.** $(5x + 1)^3 + 15(5x + 1)^2(x - 4)$
b. $15x^2(3x^2 + 4)(3 + x^3)^4 + 6x(3 + x^3)^5$
c. $-8x(1 - x^2)^3(2x + 6)^3 + 6(1 - x^2)^4(2x + 6)^2$
d. $6(x^2 - 9)^4(2x - 1)^2 + 8x(x^2 - 9)^3(2x - 1)^3$

3. It is not appropriate or necessary to use the product rule when one of the factors is a constant or when it would be easier to first determine the product of the factors and then use other rules to determine the derivative. For example, it would not be best to use the product rule for $f(x) = 3(x^2 + 1)$ or $g(x) = (x + 1)(x - 1)$.

4. $F'(x) = [b(x)][c'(x)] + [b'(x)][c(x)]$

5. **a.** 9 **d.** -36
b. -4 **e.** 22
c. -9 **f.** 671

6. $10x + y - 8 = 0$

7. **a.** $(14, -450)$
b. $(-1, 0)$

8. **a.** $3(x + 1)^2(x + 4)(x - 3)^2 + (x + 1)^3(1)(x - 3)^2 + (x + 1)^3(x + 4)[2(x - 3)]$
b. $2x(3x^2 + 4)^2(3 - x^3)^4 + x^2[2(3x^2 + 4)(6x)](3 - x^3)^4 + x^2(3x^2 + 4)^2 \times [4(3 - x^3)^3(-3x^2)]$

9. -4.84 L/h

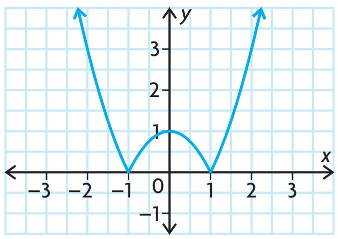
10. -30; Determine the point of tangency, and then find the negative reciprocal of the slope of the tangent. Use this information to find the equation of the normal.

11. **a.** $f'(x) = g_1'(x)g_2(x)g_3(x) \ldots g_{n-1}(x)g_n(x) + g_1(x)g_2'(x)g_3(x) \ldots g_{n-1}(x)g_n(x) + g_1(x)g_2(x)g_3'(x) \ldots g_{n-1}(x)g_n(x) + \ldots + g_1(x)g_2(x)g_3(x) \ldots g_{n-1}(x)g_n'(x)$

b. $\frac{n(n+1)}{2}$

12. $y = 3x^2 + 6x - 5$

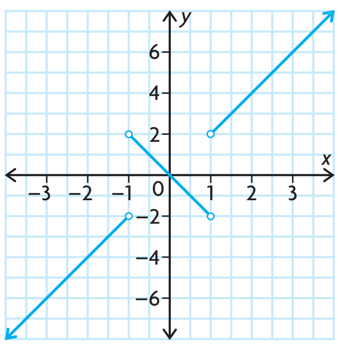
13.



a. $x = 1$ or $x = -1$

b. $f'(x) = 2x,\ x < -1$ *or* $x > 1$

$f'(x) = -2x,\ -1 < x < 1$



c. $f'(-2) = -4, f'(0) = 0, f'(3) = 6$

14. $y = \frac{16}{x^2} - 1$

$\frac{dy}{dx} = -\frac{32}{x^3}$

Slope of this line is 4.

$-\frac{32}{x^3} = 4$

$x = -2$

$y = 3$

Point is at $(-2, 3)$.

Find intersection of line and curve:

$y = 4x + 11$

Substitute,

$4x + 11 = \frac{16}{x^2} - 1$

$4x^3 + 11x^2 - 16 - x^2$ or

$4x^3 + 12x^2 - 16 = 0$

Let $x = -2$

$\text{RS} = 4(-2) + 12(-2)^2 - 16$

$= 0$

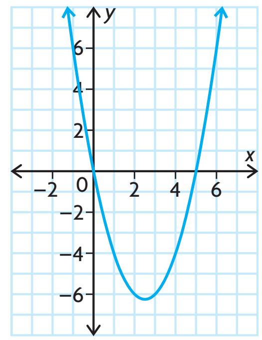
Since $x = -2$ satisfies the equation, therefore it is a solution.

When $x = -2, y = 4(-2) + 11 = 3$.

Intersection point is $(-2, 3)$. Therefore, the line is tangent to the curve.

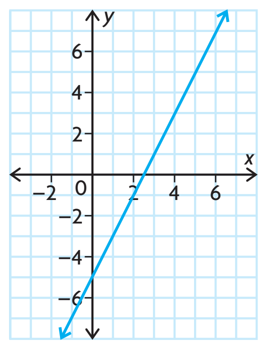
Mid-Chapter Review, pp. 92–93

1. **a.**



b. $f'(0) = -5, f'(1) = -3,$
$f'(2) = -1, f'(3) = 1, f'(4) = 3,$
$f'(5) = 5$

c.



d. $f(x)$ is quadratic; $f'(x)$ is linear.

2. **a.** 6

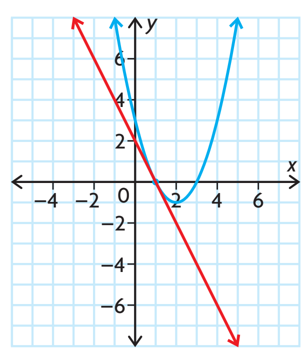
b. $4x$

c. $\frac{-5}{(x+5)^2}$

d. $\frac{1}{2\sqrt{x-2}}$

3. **a.** $y = -2x + 2$

b.



4. **a.** $24x^3$

b. $\frac{5}{\sqrt{x}}$

c. $-\frac{6}{x^4}$

d. $5 - \frac{6}{x^3}$

e. $242t + 22$

f. $\frac{1}{x^2}$

5. $y = x - \frac{3}{8}$

6. **a.** $8x - 7$

b. $-6x^2 + 8x + 5$

c. $-\frac{10}{x^3} + \frac{9}{x^4}$

d. $\frac{1}{2x^{\frac{1}{2}}} + \frac{1}{3x^{\frac{2}{3}}}$

e. $-\frac{14}{x^3} - \frac{3}{2x^{\frac{1}{2}}}$

f. $\frac{4}{x^2} + 5$

7. **a.** $y = 7$

b. $y = -\frac{1}{3}x$

c. $y = -128x + 297$

8. **a.** $48x^3 - 81x^2 + 40x - 45$

b. $-36t^2 - 50t + 39$

c. $24x^3 + 24x^2 - 78x - 36$

d. $-162x^2 + 216x^5 - 72x^8$

9. $76x - y - 28 = 0$

10. $(3, 8)$

11. $10x - 8$

12. **a.** $\frac{500}{9}$ L

b. $-\frac{200}{27}$ L/min

c. $-\frac{200}{27}$ L/min

13. **a.** $\frac{1900}{3}\pi\ \text{cm}^3/\text{cm}$

b. $256\ \pi\ \text{cm}^3/\text{cm}$

14. This statement is always true. A cubic polynomial function will have the form $f(x) = ax^3 + bx^2 + cx + d, a \neq 0$. So, the derivative of this cubic is $f'(x) = 3ax^2 + 2bx + c$ and since $3a \neq 0$, this derivative is a quadratic polynomial function. For example, if $f(x) = x^3 + x^2 + 1$, we get $f'(x) = 3x^2 + 2x$, and if $f(x) = 2x^3 + 3x^2 + 6x + 2$, we get $f'(x) = 6x^2 + 6x + 6$.

15. $y = \frac{x^{2a+3b}}{x^{a-b}}, a, b \in I$

Simplifying,

$y = x^{2a+3b-(a-b)} = x^{a+4b}$

Then,

$y'(a + 4b)^{a+4b-1}$

16. **a.** -188

b. $f'(3)$ is the slope of the tangent line to $f(x)$ at $x = 3$ and the rate of change in the value of $f(x)$ with respect to x at $x = 3$.

17. a. 100 bacteria
b. 1200 bacteria
c. 370 bacteria/h

18. $C'(t) = -\frac{100}{t^2}$; The values of the derivative are the rates of change of the percent with respect to time at 5, 50, and 100 min. The percent of carbon dioxide that is released per unit of time from the soft drink is decreasing. The soft drink is getting flat.

Section 2.4, pp. 97–98

1. For x, a, b real numbers,
$x^a x^b = x^{a+b}$
For example,
$x^9 x^{-6} = x^3$
Also,
$(x^a)^b = x^{ab}$
For example,
$(x^2)^3 = x^6$
Also,
$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$
For example,
$\frac{x^5}{x^3} = x^2$

2.

Function	Rewrite	Differentiate and Simplify, if Necessary
$f(x) = \frac{x^2 + 3x}{x}$, $x \neq 0$	$f(x) = x + 3$	$f'(x) = 1$
$g(x) = \frac{3x^{\frac{5}{3}}}{x}$, $x \neq 0$	$g(x) = 3x^{\frac{2}{3}}$	$g'(x) = 2x^{-\frac{1}{3}}$
$h(x) = \frac{1}{10x^5}$, $x \neq 0$	$h(x) = \frac{1}{10}x^{-5}$	$h'(x) = \frac{-1}{2}x^{-6}$
$y = \frac{8x^3 + 6x}{2x}$, $x \neq 0$	$y = 4x^2 + 3$	$\frac{dy}{dx} = 8x$
$s = \frac{t^2 - 9}{t - 3}$, $t \neq 3$	$s = t + 3$	$\frac{ds}{dt} = 1$

3. In the previous problem, all of these rational examples could be differentiated via the power rule after a minor algebraic simplification. A second approach would be to rewrite a rational example
$h(x) = \frac{f(x)}{g(x)}$
using the exponent rules as
$h(x) = f(x)(g(x))^{-1}$, and then apply the product rule for differentiation (together with the power of a function rule) to find $h'(x)$. A third (an perhaps easiest) approach would be to just apply the quotient rule to find $h'(x)$.

4. a. $\frac{1}{(x + 1)^2}$
b. $\frac{13}{(t + 5)^2}$
c. $\frac{2x^4 - 3x^2}{(2x^2 - 1)^2}$
d. $\frac{-2x}{(x^2 + 3)^2}$
e. $\frac{5x^2 + 6x + 5}{(1 - x^2)^2}$
f. $\frac{x^2 + 4x - 3}{(x^2 + 3)^2}$

5. a. $\frac{13}{4}$
b. $\frac{7}{25}$
c. $\frac{200}{841}$
d. $-\frac{7}{3}$

6. -9

7. $\left(9, \frac{27}{5}\right)$ and $\left(-1, \frac{3}{5}\right)$

8. Since $(x + 2)^2$ is positive or zero for all $x \in \mathbf{R}$, $\frac{8}{(x + 2)^2} > 0$ for $x \neq -2$. Therefore, tangents to the graph of $f(x) = \frac{5x + 2}{x + 2}$ do not have a negative slope.

9. a. (0, 0) and (8, 32)
b. no horizontal tangents

10. 75.4 bacteria per hour at $t = 1$ and 63.1 bacteria per hour at $t = 2$

11. $5x - 12y - 4 = 0$

12. a. 20 m
b. $\frac{10}{9}$ m/s

13. a. i. 1 cm
ii. 1 s
iii. 0.25 cm/s
b. No, the radius will never reach 2 cm because $y = 2$ is a horizontal asymptote of the graph of the function. Therefore, the radius approaches but never equals 2 cm.

14. $a = 1, b = 0$

15. 1.87 h

16. 2.83 s

17. $ad - bc > 0$

Section 2.5, pp. 105–106

1. a. 0
b. 0
c. -1
d. $\sqrt{15}$
e. $\sqrt{x^2 - 1}$
f. $x - 1$

2. a. $(f \circ g) = x$,
$(g \circ f) = |x|$,
$\{x \geq 0\}, \{x \in \mathbf{R}\}$; not equal
b. $(f \circ g) = \frac{1}{(x^2 + 1)}$,
$(g \circ f) = \left(\frac{1}{x^2}\right) + 1$,
$\{x \neq 0\}, \{x \in \mathbf{R}\}$; not equal
c. $(f \circ g) = \frac{1}{\sqrt{x + 2}}$,
$(g \circ f) = \sqrt{\frac{1}{x} + 2}$,
$\{x > -2\}, \left\{x \leq -\frac{1}{2}, x > 0\right\}$; not equal

3. If $f(x)$ and $g(x)$ are two differentiable functions of x, and
$h(x) = (f \circ g)(x)$
$= f(g(x))$
is the composition of these two functions, then $h'(x) = f'(g(x)) \times g'(x)$. This is known as the "chain rule" for differentiation of composite functions. For example, if $f(x) = x^{10}$ and $g(x) = x^2 + 3x + 5$, then $h(x) = (x^2 + 3x + 5)^{10}$, and so
$h'(x) = f'(g(x)) \times g'(x)$
$= 10(x^2 + 3x + 5)^9(2x + 3)$
As another example, if $f(x) = x^{\frac{2}{3}}$ and $g(x) = x^2 + 1$, then $h(x) = (x^2 + 1)^{\frac{2}{3}}$, and so $h'(x) = \frac{2}{3}(x^2 + 1)^{-\frac{1}{3}}(2x)$.

4. a. $8(2x + 3)^3$
b. $6x(x^2 - 4)^2$
c. $4(2x^2 + 3x - 5)^3(4x + 3)$
d. $-6x(\pi^2 - x^2)^2$
e. $\frac{x}{\sqrt{x^2 - 3}}$
f. $\frac{-10x}{(x^2 - 16)^6}$

5. a. $-2x^{-3}; \frac{6}{x^4}$
b. $(x + 1)^{-1}; \frac{-1}{(x + 1)^2}$
c. $(x^2 - 4)^{-1}; \frac{-2x}{(x^2 - 4)^2}$
d. $3(9 - x^2)^{-1}; \frac{6x}{(9 - x^2)^2}$
e. $(5x^2 + x)^{-1}; -\frac{10x + 1}{(5x^2 + x)^2}$
f. $(x^2 + x + 1)^{-4}; -\frac{8x + 4}{(x^2 + x + 1)^5}$

6. $h(-1) = -4$; $h'(-1) = 35$

7. $-\frac{2}{x^2}\left(\frac{1}{x} - 3\right)$

8. **a.** $(x + 4)^2(x - 3)^5(9x + 15)$
b. $6x(x^2 + 3)^2(x^3 + 3)(2x^3 + 3x + 3)$
c. $\dfrac{-2x^2 + 6x + 2}{(x^2 + 1)^2}$
d. $15x^2(3x - 5)(x - 1)$
e. $4x^3(1 - 4x^2)^2(1 - 10x^2)$
f. $\dfrac{48x(x^2 - 3)^3}{(x^2 + 3)^5}$

9. **a.** $\dfrac{91}{36}$ **b.** $-\dfrac{5\sqrt[3]{2}}{24\pi}$

10. $x = 0$ or $x = 1$

11. $\dfrac{1}{4}$

12. $60x - y - 119 = 0$

13. **a.** 52 **b.** 54 **c.** 320 **d.** 78

14. -6

15. 2222 L/min

16. 2.75 m/s

17. **a.** $p'(x)q(x)r(x) + p(x)q'(x)r(x) + p(x)q(x)r'(x)$
b. -344

18. $\dfrac{dy}{dx} = 3(x^2 + x - 2)^2(2x + 1)$
At the point (1, 3), slope of the tangent will be $3(1 + 1 - 2)^2(2 + 1) = 0$.
Equation of the tangent at (1, 3) is
$y = 3$
$(x^2 + x - 2)^3 + 3 = 3$
$(x + 2)^3(x - 1)^3 = 0$
$x = -2$ or $x = 1$
Since -2 and 1 are both triple roots, the line with equation $y = 3$ is also a tangent at $(-2, 3)$.

19. $-\dfrac{2x(x^2 + 3x - 1)(1 - x)^2}{(1 + x)^4}$

Review Exercise, pp. 110–113

1. To find the derivative $f'(x)$, the limit $f'(x) = \lim_{h \to 0} \dfrac{f(x + h) - f(x)}{h}$ must be computed, provided it exists. If this limit does not exist, then the derivative of $f(x)$ does not exist at this particular value of x. As an alternative to this limit, we could also find $f'(x)$ from the definition by computing the equivalent limit $f'(x) = \lim_{z \to x} \dfrac{f(z) - f(x)}{z - x}$. These two limits are seen to be equivalent by substituting $z = x + h$.

2. **a.** $4x - 5$ **c.** $\dfrac{4}{(4 - x)^2}$
b. $\dfrac{1}{2\sqrt{x - 6}}$

3. **a.** $2x - 5$
b. $\dfrac{3}{4x^{\frac{1}{4}}}$
c. $-\dfrac{28}{3x^5}$
d. $-\dfrac{2x}{(x^2 + 5)^2}$
e. $\dfrac{12x}{(3 - x^2)^3}$
f. $\dfrac{7x + 2}{\sqrt{7x^2 + 4x + 1}}$

4. **a.** $2 + \dfrac{2}{x^3}$
b. $\dfrac{\sqrt{x}}{2}(7x^2 - 3)$
c. $-\dfrac{5}{(3x - 5)^2}$
d. $\dfrac{3x - 1}{2\sqrt{x - 1}}$
e. $-\dfrac{1}{3\sqrt{x}(\sqrt{x} + 2)^{\frac{5}{3}}}$
f. 1

5. **a.** $20x^3(2x - 5)^5(x - 1)$
b. $\dfrac{x^2}{\sqrt{x^2 + 1}} + \sqrt{x^2 + 1}$
c. $\dfrac{(2x - 5)^3(2x + 23)}{(x + 1)^4}$
d. $\dfrac{318(10x - 1)^5}{(3x + 5)^7}$
e. $(x - 2)^2(x^2 + 9)^3 \times (11x^2 - 16x + 27)$
f. $-\dfrac{6(1 - x^2)^2(x^2 + 6x + 1)}{(6 + 2x)^4}$

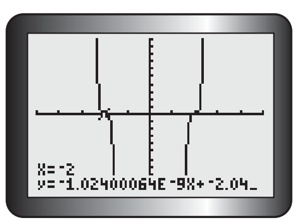
6. **a.** $f'(x^2) \times 2x$
b. $2xf'(x) + 2f(x)$

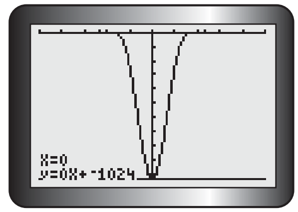
7. **a.** $-\dfrac{184}{9}$ **b.** $-\dfrac{25}{289}$ **c.** $-\dfrac{8}{5}$

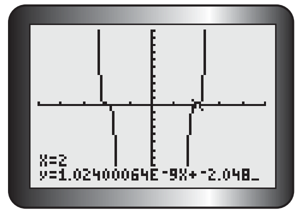
8. $-\dfrac{2}{3}$

9. $x = 2 \pm 2\sqrt{2}$;
$x = 5, x = -1$

10. **a.** **i.** $x = 0, x = \pm 2$
ii. $x = 0, x = \pm 1, x = \pm\dfrac{\sqrt{3}}{3}$
b. **i.**







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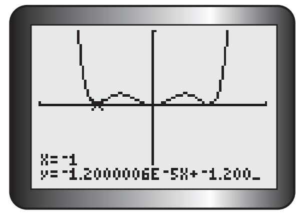
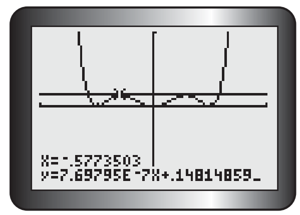
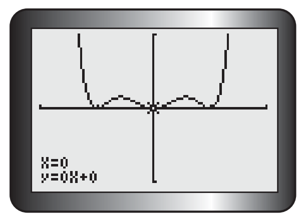
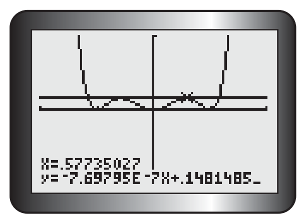
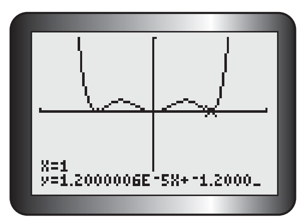








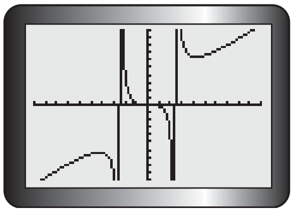


11. **a.** $160x - y + 16 = 0$
b. $60x + y - 61 = 0$

12. $5x - y - 7 = 0$

13. $(2, 8)$; $b = -8$

14. **a.**



b. $y = 0$, $y = 6.36$, $y = -6.36$

c. $(0, 0)$, $\left(3\sqrt{2}, \frac{9\sqrt{2}}{2}\right)$, $\left(-3\sqrt{2}, -\frac{9\sqrt{2}}{2}\right)$

d. -14

15. **a.** $\sqrt[3]{50}$
b. 1

16. **a.** When $t = 10$, 9; when $t = 15$, 19
b. At $t = 10$, the number of words memorized is increasing by 1.7 words/min. At $t = 15$, the number of words memorized is increasing by 2.325 words/min.

17. **a.** $\frac{30t}{(9 + t^2)^{\frac{1}{2}}}$
b. No; since $t > 0$, the derivative is always positive, meaning that the rate of change in the cashier's productivity is always increasing. However, these increases must be small, since, according to the model, the cashier's productivity can never exceed 20.

18. **a.** $x^2 + 40$
b. 6 gloves/week

19. **a.** $750 - \frac{x}{3} - 2x^2$
b. $546.67

20. $-\frac{5}{4}$

21. **a.** $B(0) = 500$, $B(30) = 320$
b. $B'(0) = 0$, $B'(30) = -12$
c. $B(0) =$ blood sugar level with no insulin
$B(30) =$ blood sugar level with 30 mg of insulin
$B'(0) =$ rate of change in blood sugar level with no insulin
$B'(30) =$ rate of change in blood sugar level with 30 mg of insulin
d. $B'(50) = -20$, $B(50) = 0$
$B'(50) = -20$ means that the patient's blood sugar level is decreasing at 20 units/mg of insulin 1 h after 50 mg of insulin is injected.
$B(50) = 0$ means that the patient's blood sugar level is *zero* 1 h after 50 mg of insulin is injected. These values are not logical because a person's blood sugar level can never reach zero and continue to decrease.

22. **a.** $f(x)$ is not differentiable at $x = 1$ because it is not defined there (vertical asymptote at $x = 1$).
b. $g(x)$ is not differentiable at $x = 1$ because it is not defined there (hole at $x = 1$).
c. The graph has a cusp at $(2, 0)$ but is differentiable at $x = 1$.
d. The graph has a corner at $x = 1$, so $m(x)$ is not differentiable at $x = 1$.

23. **a.** $f(x)$ is not defined at $x = 0$ and $x = 0.25$. The graph has vertical asymptotes at $x = 0$ and $x = 0.25$. Therefore, $f(x)$ is not differentiable at $x = 0$ and $x = 0.25$.
b. $f(x)$ is not defined at $x = 3$ and $x = -3$. At $x = -3$, the graph has a vertical asymptote and at $x = 3$ it has a hole. Therefore, $f(x)$ is not differentiable at $x = 3$ and $x = -3$.
c. $f(x)$ is not defined for $1 < x < 6$. Therefore, $f(x)$ is not differentiable for $1 < x < 6$.

24. $\frac{25}{(t + 1)^2}$

25. Answers may vary. For example:
$f(x) = 2x + 3$

$$y = \frac{1}{2x + 3}$$
$$y' = \frac{(2x + 3)(0) - (1)(2)}{(2x + 3)^2}$$
$$= -\frac{2}{(2x + 3)^2}$$

$f(x) = 5x + 10$

$$y = \frac{1}{5x + 10}$$
$$y' = \frac{(5x + 10)(0) - (1)(5)}{(5x + 10)^2}$$
$$= -\frac{5}{(5x + 10)^2}$$

Rule: If $f(x) = ax + b$ and $y = \frac{1}{f(x)}$, then

$$y' = \frac{-a}{(ax + b)^2}$$
$$y' = \lim_{h \to 0} \frac{1}{h}\left[\frac{1}{a(x + h) + b} - \frac{1}{ax + b}\right]$$
$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{ax + b - [a(x + h) + b]}{[a(x + h) + b](ax + h)}\right]$$
$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{-ah}{[a(x + h) + b](ax + b)}\right]$$
$$= \lim_{h \to 0}\left[\frac{-a}{[a(x + h) + b](ax + b)}\right]$$
$$= \frac{-a}{(ax + b)^2}$$

26. **a.** $y = u + 5u^{-1}$, $u = 2 \times -3$
b. $2(1 - 5(2x - 3)^{-2})$

27. **a.** $y = \sqrt{u} + 5u$, $u = 2 \times -3$
b. $(2x - 3)^{-\frac{1}{2}} + 10$

28. **a.** $6(2x - 5)^2(3x^2 + 4)^4 \times (13x^2 - 25x + 4)$
b. $8x^2(4x^2 + 2x - 3)^4 (52x^2 + 16x - 9)$
c. $2(5 + x)(4 - 7x^3)^5 \times (4 - 315x^2 - 70x^3)$
d. $\frac{6(-9x + 7)}{(3x + 5)^5}$
e. $\frac{2(2x^2 - 5)^2(4x^2 + 48x + 5)}{(x + 8)^3}$
f. $\frac{-3x^3(7x - 16)}{(4x - 8)^{\frac{3}{2}}}$
g. $8\left(\frac{2x + 5}{6 - x^2}\right)^3\left(\frac{(x + 2)(x + 3)}{(6 - x^2)^2}\right)$
h. $-9(4x + x^2)^{-10}(4 + 2x)$

29. $a = -4$, $b = 32$, $c = 0$

30. **a.** $-3t^2 + 5$
b. -7000 ants/h
c. 75 000 ants
d. 9.27 h

Chapter 2 Test, p. 114

1. You need to use the chain rule when the derivative for a given function cannot be found using the sum, difference, product, or quotient rules or when writing the function in a form that would allow the use of these rules is tedious. The chain rule is used when a given function is a composition of two or more functions.

2. f is the blue graph (it's cubic). f' is the red graph (it is quadratic). The derivative of a polynomial function has degree one less than the derivative of the function. Since the red graph is a quadratic (degree 2) and the blue graph is cubic (degree 3), the blue graph is f and the red graph is f'.

3. $1 - 2x$

4. a. $x^2 + 15x^{-6}$

b. $60(2x - 9)^4$

c. $-x^{-\frac{3}{2}} + \frac{1}{\sqrt{3}} + 2x^{-\frac{2}{3}}$

d. $\frac{5(x^2 + 6)^4(3x^2 + 8x - 18)}{(3x + 4)^6}$

e. $2x(6x^2 - 7)^{-\frac{2}{3}}(8x^2 - 7)$

f. $\frac{4x^5 - 18x + 8}{x^5}$

5. 14

6. $-\frac{40}{3}$

7. $60x + y - 61 = 0$

8. $\frac{75}{32}$ ppm/year

9. $\left(-\frac{1}{4}, \frac{1}{256}\right)$

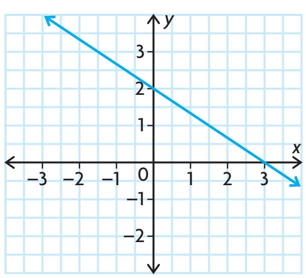
10. $\left(-\frac{1}{3}, \frac{32}{27}\right)$, (1, 0)

11. $a = 1, b = -1$

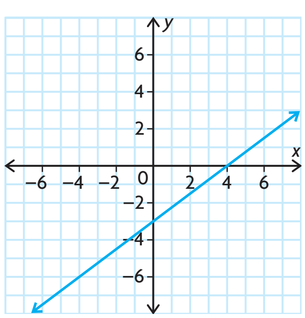
Chapter 3

Review of Prerequisite Skills, pp. 116–117

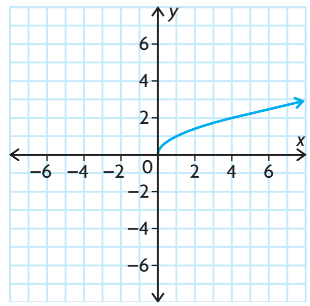
1. a.



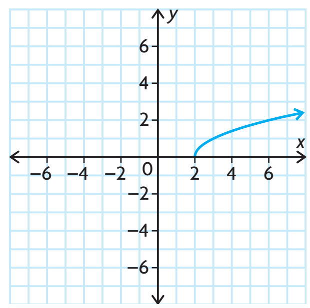
b.



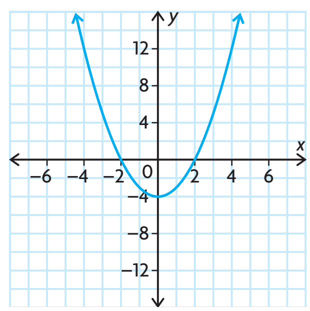
c.



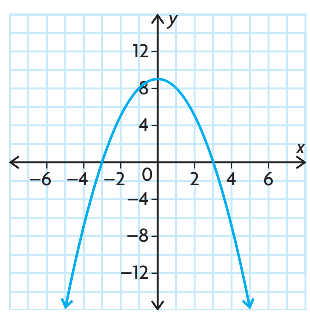
d.



e.



f.



2. a. $x = \frac{14}{5}$

b. $x = -13$

c. $t = 3$ or $t = 1$

d. $t = -\frac{1}{2}$ or $t = 3$

e. $t = 2$ or $t = 6$

f. $x = 0$ or $x = -3$ or $x = 1$

g. $x = 0$ or $x = 4$

h. $t = -3$ or $t = \frac{1}{2}$ or $t = -\frac{1}{2}$

i. $t = \pm\frac{3}{2}$ or $t = \pm 1$

3. a. $x > 3$

b. $x < 0$ or $x > 3$

c. $0 < x < 4$

4. a. 25 cm^2 c. 49π cm^2

b. 48 cm^2 d. 36π cm^2

5. a. $SA = 56\pi$ cm^2, $V = 48\pi$ cm^3

b. $h = 6$ cm, $SA = 80\pi$ cm^2

c. $r = 6$ cm, $SA = 144\pi$ cm^2

d. $h = 7$ cm, $V = 175\pi$ cm^3

6. a. $SA = 54$ cm^2, $V = 27$ cm^3

b. $SA = 30$ cm^2, $V = 5\sqrt{5}$ cm^3

c. $SA = 72$ cm^2, $V = 24\sqrt{3}$ cm^3

d. $SA = 24k^2$ cm^2, $V = 8k^3$ cm^3

7. a. $(3, \infty)$ d. $[-5, \infty)$

b. $(-\infty, -2]$ e. $(-2, 8]$

c. $(-\infty, 0)$ f. $(-4, 4)$

8. a. $\{x \in \mathbf{R} \mid x > 5\}$

b. $\{x \in \mathbf{R} \mid x \leq 1\}$

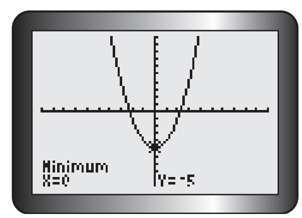
c. $\{x \in \mathbf{R}\}$

d. $\{x \in \mathbf{R} \mid -10 \leq x \leq 12\}$

e. $\{x \in \mathbf{R} \mid -1 < x < 3\}$

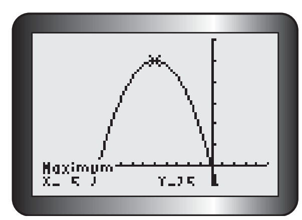
f. $\{x \in \mathbf{R} \mid 2 \leq x < 20\}$

9. a.



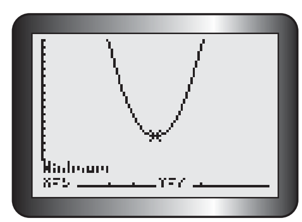
The function has a minimum value of -5 and no maximum value.

b.



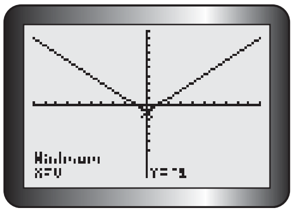
The function has a maximum value of 25 and no minimum value.

c.



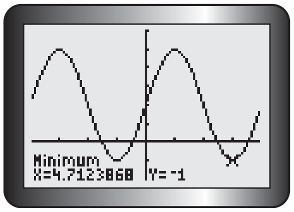
The function has a minimum value of 7 and no maximum value.

d.

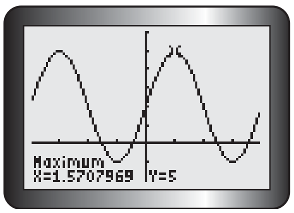


The function has a minimum value of -1 and no maximum value.

e.

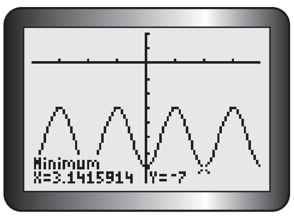


The function has a minimum value of -1.

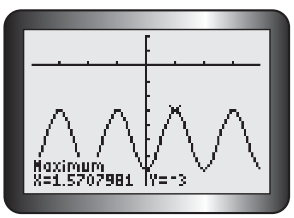


The function has a maximum value of 5.

f.



The function has a minimum value of -7.



The function has a maximum value of -3.

Section 3.1, pp. 127–129

1. At $t = 1$, the velocity is positive; this means that the object is moving in whatever is the positive direction for the scenario. At $t = 5$, the velocity is negative; this means that the object is moving in whatever is the negative direction for the scenario.

2. **a.** $y'' = 90x^8 + 90x^4$

b. $f''(x) = -\frac{1}{4}x^{-\frac{3}{2}}$

c. $y'' = 2$

d. $h''(x) = 36x^2 - 24x - 6$

e. $y'' = \frac{3}{\sqrt{x}} - \frac{6}{x^4}$

f. $f''(x) = -\frac{4}{(x+1)^3}$

g. $y'' = 2 + \frac{6}{x^4}$

h. $g''(x) = -\frac{9}{4(3x-6)^{\frac{3}{2}}}$

i. $y'' = 48x + 96$

j. $h''(x) = \frac{10}{9x^{\frac{1}{3}}}$

3. **a.** $v(t) = 10t - 3,$
$a(t) = 10$

b. $v(t) = 6t^2 + 36,$
$a(t) = 12t$

c. $v(t) = 1 - 6t^{-2},$
$a(t) = 12t^{-3}$

d. $v(t) = 2(t - 3),$
$a(t) = 2$

e. $v(t) = \frac{1}{2\sqrt{t+1}}$

$a(t) = -\frac{1}{4(\sqrt{t+1})^3}$

f. $v(t) = \frac{27}{(t+3)^2},$

$a(t) = -54(t+3)^{-3}$

4. **a.** **i.** $t = 3$
ii. $1 < t < 3$
iii. $3 < t < 5$

b. **i.** $t = 3, t = 7$
ii. $1 < t < 3, 7 < t$
iii. $3 < t < 7$

5. **a.** $v(t) = t^2 - 4t + 3,$
$a(t) = 2t - 4$

b. at $t = 1$ and $t = 3$

c. $t > 3$

6. **a.** For $t = 1$, moving in a positive direction.
For $t = 4$, moving in a negative direction.

b. For $t = 1$, the object is stationary.
For $t = 4$, the object is moving in a positive direction.

c. For $t = 1$, the object is moving in a negative direction.
For $t = 4$, the object is moving in a positive direction.

7. **a.** $v(t) = 2t - 6$

b. $t = 3$ s

8. **a.** $t = 4$ s

b. $s(4) = 80$ m

9. **a.** $v(5) = 3$ m/s

b. $a(5) = 2$ m/s^2

10. **a.** $v(t) = \frac{35}{2}t^{\frac{3}{2}} - \frac{7}{2}t^{\frac{5}{2}},$

$a(t) = \frac{105}{4}t^{\frac{1}{2}} - \frac{35}{4}t^{\frac{3}{2}}$

b. 5 s

c. 5 s

d. $0 < t < 3$ s

e. after 7 s

11. **a.** 25 m/s

b. 31.25 m

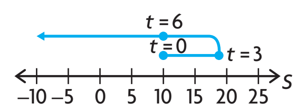
c. $t = 5$ s; -25 m/s

12. **a.** $v(8) = 98$ m/s,
$a(8) = 12$ m/s^2

b. 38 m/s

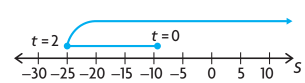
13. **a.** $s = 10 + 6t - t^2$
$v = 6 - 2t$
$= 2(3 - t)$
$a = -2$

The object moves to the right from its initial position of 10 m from the origin, 0, to the 19 m mark, slowing down at a rate of 2 m/s^2. It stops at the 19 m mark, then moves to the left, accelerating at 2 m/s^2 as it goes. It passes the origin after $(3 + \sqrt{19})$ s.



b. $s = t^3 - 12t - 9$
$v = 3t^2 - 12$
$= 3(t^2 - 4)$
$= 3(t - 2)(t + 2)$
$a = 6t$

The object begins at 9 m to the left of the origin, 0, and slows down to a stop after 2 s when it is 25 m to the left of the origin. Then, the object moves to the right, accelerating at faster rates as time increases. It passes the origin just before 4 s (approximately 3.7915) and continues to accelerate as time goes by.



14. $t = 1$ s; away

15. a. $s(t) = kt^2 + (6k^2 - 10k)t + 2k$

$v(t) = 2kt + (6k^2 - 10k)$

$a(t) = 2k + 0$

$= 2k$

Since $k \neq 0$ and $k \in \mathbf{R}$, then $a(t) = 2k \neq 0$ and an element of the real numbers. Therefore, the acceleration is constant.

b. $t = 5 - 3k$, $-9k^3 + 30k^2 - 23k$.

16. a. The acceleration is continuous at $t = 0$ if $\lim_{t \to 0} a(t) = a(0)$.

For $t \geq 0$,

$$s(t) = \frac{t^3}{t^2 + 1}$$

$$\text{and } v(t) = \frac{3t^2(t^2 + 1) - 2t(t^3)}{(t^2 + 1)^2}$$

$$= \frac{t^4 + 3t^2}{(t^2 + 1)^2}$$

$$\text{and } a(t) = \frac{(4t^3 + 6t)(t^2 + 1)^2}{(t^2 + 1)^2} - \frac{2(t^2 + 1)(2t)(t^4 + 3t^2)}{(t^2 + 1)^2}$$

$$= \frac{(4t^3 + 6t)(t^2 + 1)}{(t^2 + 1)^3} - \frac{4t(t^4 + 3t^2)}{(t^2 + 1)^3}$$

$$= \frac{4t^5 + 6t^3 + 4t^3}{(t^2 + 1)^3} + \frac{6t - 4t^5 - 12t^3}{(t^2 + 1)^3}$$

$$= \frac{-2t^3 + 6t}{(t^2 + 1)^3}$$

Therefore,

$$a(t) = \begin{cases} 0, \text{ if } t < 0 \\ \dfrac{-2t^3 + 6t}{(t^2 + 1)^3}, \text{ if } t \geq 0 \end{cases}$$

and

$$v(t) = \begin{cases} 0, \text{ if } t < 0 \\ \dfrac{t^4 + 3t^2}{(t^2 + 1)^2}, \text{ if } t \geq 0 \end{cases}$$

$$\lim_{t \to 0^-} a(t) = 0, \quad \lim_{t \to 0^+} a(t) = \frac{0}{1} = 0$$

Thus, $\lim_{t \to 0} a(t) = 0$.

Also, $a(0) = \dfrac{0}{1} = 0$

Therefore, $\lim_{t \to 0} a(t) = a(0)$.

Thus, the acceleration is continuous at $t = 0$.

b. velocity approaches 1, acceleration approaches 0

17. $v = \sqrt{b^2 + 2gs}$

$$v = (b^2 + 2gs)^{\frac{1}{2}}$$

$$\frac{dv}{dt} = \frac{1}{2}(b^2 + 2gs)^{-\frac{1}{2}} \times \left(0 + 2g\frac{ds}{dt}\right)$$

$$a = \frac{1}{2v} \times 2gv$$

$$a = g$$

Since g is a constant, a is a constant, as required.

Note: $\dfrac{ds}{dt} = v$

$\dfrac{dv}{dt} = a$

18. $F = m_0 \dfrac{d}{dt}\left(\dfrac{v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}}\right)$

Using the quotient rule,

$$= \frac{m_0 \frac{dv}{dt}\left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}}{1 - \frac{v^2}{c^2}} - \frac{\frac{1}{2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\left(-\frac{2v\frac{dv}{dt}}{c^2}\right) \times v}{1 - \frac{v^2}{c^2}}$$

Since $\dfrac{dv}{dt} = a$,

$$= \frac{m_0\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}\left[a\left(1 - \frac{v^2}{c^2}\right) + \frac{v^2 a}{c^2}\right]}{1 - \frac{v^2}{c^2}}$$

$$= \frac{m_0\left[\frac{ac^2 - av^2}{c^2} + \frac{v^2 a}{c^2}\right]}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$= \frac{m_0 ac^2}{c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

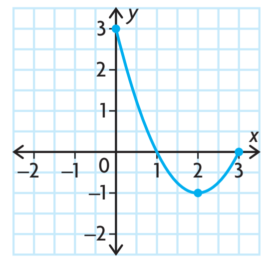
$$= \frac{m_0 a}{\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}, \text{ as required.}$$

Section 3.2, pp. 135–138

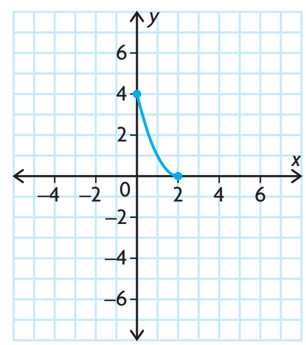
1. a. The algorithm can be used; the function is continuous.
   b. The algorithm cannot be used; the function is discontinuous at $x = 2$.
   c. The algorithm cannot be used; the function is discontinuous at $x = 2$.
   d. The algorithm can be used; the function is continuous on the given domain.

2. a. max: 8, min: -12
   b. max: 30, min: -5
   c. max: 100, min: -100
   d. max: 30, min: -20

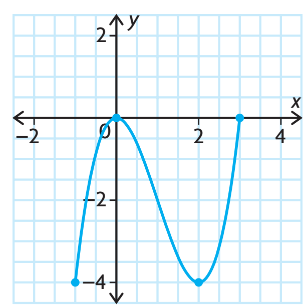
3. a. max is 3 at $x = 0$, min is -1 at $x = 2$



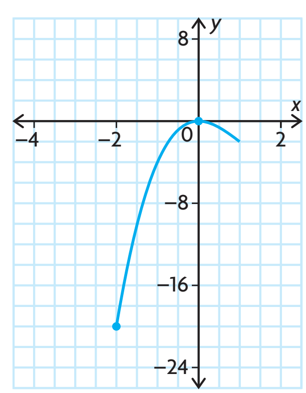
b. max is 4 at $x = 0$, min is 0 at $x = 2$



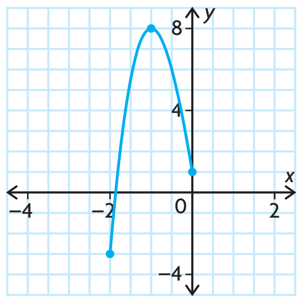
c. min is -4 at $x = -1, 2$, max is 0 at $x = 0, 3$



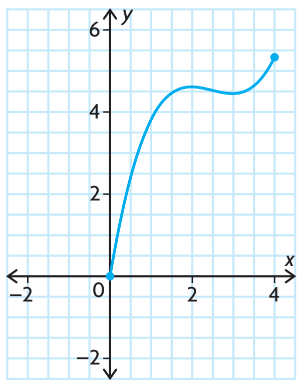
d. max is 0 at $x = 0$, min is -20 at $x = -2$



e. max is 8 at $x = -1$,
min is -3 at $x = -2$

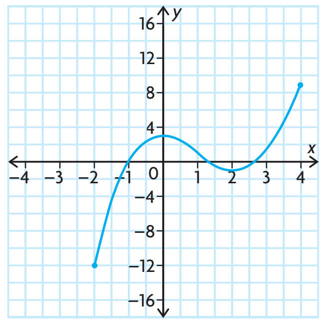


f. max is $\frac{16}{3}$ at $x = 4$,
min is 0 at $x = 0$



4. **a.** min value of 4 when $x = 2$,
max value of 10.4 when $x = 10$
b. min value of 3 when $x = 9$,
max value of 4 when $x = 4$
c. max value of 1 when $x = 1$,
min value of $\frac{1}{2}$ when $x = 0, 2$
d. min value of -169 when $x = 3$,
max value of 47 when $x = -3$
e. max value of 2 when $x = 1$,
min value of -2 when $x = -1$
f. min value of 0.94 when $x = 4$,
max value of 1.6 when $x = 2$
5. **a.** max velocity is $\frac{4}{3}$ m/s,
min velocity is $\frac{4}{5}$ m/s
b. min velocity is 0 m/s, no maximum velocity, but $v(t) \to 4$ as $t \to \infty$
6. 20 bacteria/cm^3
7. **a.** 80 km/h
b. 50 km/h
c. $0 \le v < 80$
d. $80 < v \le 100$
8. min concentration $\doteq 0.006\,25$
max concentration $\doteq 0.0083$
9. 0.05 years or approximately 18 days
10. 70 km/h; \$80.50
11. absolute max value $= 42$,
absolute min value $= 10$

12. **a.**



b. $-2 \le x \le 4$
c. increasing: $-2 \le x < 0$
$2 < x \le 4$
decreasing: $0 < x < 2$
13. Absolute max: Compare all local maxima and values of $f(a)$ and $f(b)$ when the domain of $f(x)$ is $a \le x \le b$. The one with the highest value is the absolute maximum.
Absolute min: We need to consider all local minima and the value of $f(a)$ and $f(b)$ when the domain of $f(x)$ is $a \le x \le b$. Compare them, and the one with the lowest value is the absolute minimum.
You need to check the endpoints because they are not necessarily critical points.
14. 245 units
15. 300 units

Mid-Chapter Review, pp. 139–140

1. **a.** $h''(x) = 36x^2 - 24x - 6$
b. $f''(x) = 48x - 120$
c. $y'' = \dfrac{30}{(x+3)^3}$
d. $g''(x) = -\dfrac{x^2}{(x^2+1)^{\frac{3}{2}}} + \dfrac{1}{(x^2+1)^{\frac{1}{2}}}$
2. **a.** 108 m
b. -45 m/s
c. -18 m/s^2
3. **a.** 6 m/s
b. $t \doteq 0.61$ s
c. $t = 1.50$ s
d. -8.67 m/s
e. -9.8 m/s^2, -9.8 m/s^2
4. **a.** Velocity is 0 m/s
Acceleration is 10 m/s^2
b. Object is stationary at time $t = \frac{1}{3}$ s and $t = 2$ s.
Before $t = \frac{1}{3}$, $v(t)$ is positive and therefore the object is moving to the right.
Between $t = \frac{1}{3}$ and $t = 2$, $v(t)$ is negative and therefore the object is moving to the left.
After $t = 2$, $v(t)$ is positive and therefore the object is moving to the right.
c. $t \doteq 1.2$ s; At that time, the object is neither accelerating nor decelerating.
5. **a.** min value is 1 when $x = 0$,
max value is 21 when $x = 2$
b. min value is 0 when $x = -2$,
max value is 25 when $x = 3$
c. min value is 0 when $x = 1$,
max value is 0.38 when $x = \sqrt{3}$
6. 30 °C
7. **a.** 105
b. 3
c. -6
d. -78
e. 3
f. 1448
g. $-\dfrac{202}{27}$
h. $-\dfrac{185}{16}$
8. -1.7 m/s^2
9. **a.** 189 m/s
b. 27 s
c. 2916 m
d. 6.2 m/s^2
10. 16 m; 4 s
11. **a.** $0 \le t \le 4.31$
b. 2.14 s
c. 22.95 m

Section 3.3, pp. 145–147

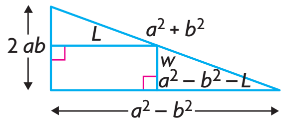
1. 25 cm by 25 cm
2. If the perimeter is fixed, then the figure will be a square.
3. 150 m by 300 m
4. height 8.8 cm, length 82.4 cm, and width 22.4 cm
5. 110 cm by 110 cm
6. 8 m by 8 m
7. 125 m by 166.67 m
8. 4 m by 6 m by 6 m
9. base 10 cm by 10 cm, height 10 cm
10. 100 square units when $5\sqrt{2}$
11. **a.** $r = 5.42$, $h = 10.84$
b. $\dfrac{h}{d} = \dfrac{1}{1}$; yes
12. **a.** 15 cm^2 when $W = 2.5$ cm and $L = 6$ cm
b. 30 cm^2 when $W = 4$ cm and $L = 7.5$ cm
c. The largest area occurs when the length and width are each equal to one-half of the sides adjacent to the right angle.
13. **a.** base is 20 cm and each side is 20 cm
b. approximately 260 000 cm^3

14. **a.** triangle side length 0.96 cm, rectangle 0.96 cm by 1.09 cm
b. Yes. All the wood would be used for the outer frame.

15. 0.36 h after the first train left the station

16. 1:02 p.m.; 3 km

17.



$$\frac{a^2 - b^2 - L}{a^2 - b^2} = \frac{W}{2ab}$$

$$W = \frac{2ab}{a^2 - b^2}(a^2 - b^2 - L)$$

$$A = LW = \frac{2ab}{a^2 - b^2}[a^2L - b^2L - L^2]$$

Let $\frac{dA}{dL} = a^2 - b^2 - 2L = 0,$

$$L = \frac{a^2 - b^2}{2} \text{ and}$$

$$W = \frac{2ab}{a^2 - b^2}\left[a^2 - b^2 - \frac{a^2 - b^2}{2}\right]$$

$$= ab$$

The hypothesis is proven.

18. Let the height be h and the radius r.

Then, $\pi r^2 h = k$, $h = \frac{k}{\pi r^2}$.

Let M represent the amount of material,

$$M = 2\pi r^2 + 2\pi rh$$

$$= 2\pi r^2 + 2\pi r\left(\frac{k}{\pi r^2}\right)$$

$$= 2\pi r^2 + \frac{2k}{r}, 0 \le r \le \infty$$

Using the max min Algorithm,

$$\frac{dM}{dr} = 4\pi r - \frac{2k}{r^2}$$

Let $\frac{dM}{dr} = 0$, $r^3 = \frac{k}{2\pi}$, $r \ne 0$ or

$$r = \left(\frac{k}{2\pi}\right)^{\frac{1}{3}}.$$

When $r \to 0$, $M \to \infty$

$r \to \infty$, $M \to \infty$

$$r = \left(\frac{k}{2\pi}\right)^{\frac{1}{3}}$$

$$d = 2\left(\frac{k}{2\pi}\right)^{\frac{1}{3}}$$

$$h = \frac{k}{\pi\left(\frac{k}{2\pi}\right)^{\frac{2}{3}}} = \frac{k}{\pi} \cdot \frac{(2\pi)^{\frac{2}{3}}}{k^{\frac{2}{3}}} = \frac{k^{\frac{1}{3}}}{\pi^{\frac{1}{3}}} \times 2^{\frac{2}{3}}$$

Min amount of material is

$$M = 2\pi\left(\frac{k}{2\pi}\right)^{\frac{2}{3}} + 2k\left(\frac{2\pi}{k}\right)^{\frac{1}{3}}.$$

Ratio $\frac{h}{d} = \frac{\left(\frac{k}{\pi}\right)^{\frac{1}{3}} \times 2^{\frac{2}{3}}}{2\left(\frac{k}{2\pi}\right)^{\frac{1}{3}}} = \frac{\left(\frac{k}{\pi}\right)^{\frac{1}{3}} \times 2^{\frac{2}{3}}}{2^{\frac{2}{3}}\left(\frac{k}{\pi}\right)^{\frac{1}{3}}} = \frac{1}{1}$

19. **a.** no cut
b. 44 cm for circle; 56 cm for square

20. $\sqrt{17}$

21. Let point A have coordinates $(a^2, 2a)$. (Note that the x-coordinate of any point on the curve is positive, but that the y-coordinate can be positive or negative. By letting the x-coordinate be a^2, we eliminate this concern.) Similarly, let B have coordinates $(b^2, 2b)$. The slope of AB is $\frac{2a - 2b}{a^2 - b^2} = \frac{2}{a + b}$.
Using the mid-point property, C has coordinates $\left(\frac{a^2 + b^2}{2}, a + b\right)$.
Since CD is parallel to the x-axis, the y-coordinate of D is also $a + b$. The slope of the tangent at D is given by $\frac{dy}{dx}$ for the expression $y^2 = 4x$.
Differentiating,

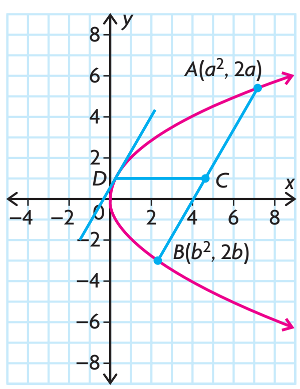
$$2y\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = \frac{2}{y}$$

And since at point D, $y = a + b$,

$$\frac{dy}{dx} = \frac{2}{a + b}$$

But this is the same as the slope of AB. Then, the tangent at D is parallel to the chord AB.



22. when P is at the point (5, 2.5)

23. $\frac{2k}{\sqrt{3}}$ by $\frac{2}{3}k^2$

Section 3.4, pp. 151–154

1. **a.** \$1.80
b. \$1.07
c. 5625 L

2. **a.** 15 terms
b. 16 terms/h
c. 20 terms/h

3. **a.** $t = 1$
b. 1.5
c.

d. The level will be a maximum.
e. The level is decreasing.

4. \$6000/h when plane is flying at 15 000 m

5. 250 m by 375 m

6. \$1100 or \$1125

7. \$22.50

8. 6 nautical miles/h

9. 20.4 m by 40.8 m by 24.0 m

10. $r = 4.3$ cm, $h = 17.2$ cm

11. **a.** \$15
b. \$12.50, \$825
c. If you increase the price, the number sold will decrease. Profit in situations like this will increase for several price increases and then it will decrease because too many customers stop buying.

12. 12.1 cm by 18.2 cm by 18.2 cm

13. \$50

14. \$81.25

15. 19 704 units

16. $P(x) = R(x) - C(x)$
Marginal Revenue $= R'(x)$.
Marginal Cost $= C'(x)$.
Now $P'(x) = R'(x) - C'(x)$.
The critical point occurs when $P'(x) = 0$.
If $R'(x) = C'(x)$, then

$$P'(x) = R'(x) - C'(x)$$
$$= 0$$

Therefore, the instantaneous rate of change in profit is 0 when the marginal revenue equals the marginal cost.

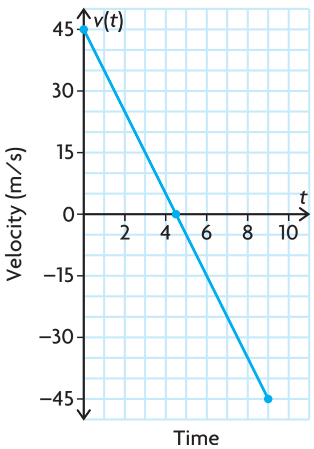
17. $r = 230$ cm and h is about 900 cm

18. 128.4 km/h

19. maximum velocity: $\frac{4}{27}r_0^3A$, radius: $\frac{2r_0}{3}$.

Review Exercise, pp. 156–159

1. $f'(x) = 4x^3 + 4x^{-5}$,
   $f''(x) = 12x^2 - 20x^{-6}$
2. $\frac{d^2y}{dx^2} = 72x^7 - 42x$
3. $v = 2t + (2t - 3)^{-\frac{1}{2}}$,
   $a = 2 - (2t - 3)^{-\frac{3}{2}}$
4. $v(t) = 1 - 5t^{-2}$,
   $a(t) = 10t^{-3}$
5. The upward velocity is positive for $0 \le t < 4.5$ s, zero for $t = 4.5$ s, and negative for $t > 4.5$ s.



6. **a.** min: -52,
   max: 0
   b. min: -65,
   max: 16
   c. min: 12,
   max: 20
7. **a.** 62 m
   b. Yes, 2 m beyond the stop sign
   c. Stop signs are located two or more metres from an intersection. Since the car only went 2 m beyond the stop sign, it is unlikely the car would hit another vehicle travelling perpendicular.
8. min is 2, max is $2 + 3\sqrt{3}$
9. 250
10. **a.** **i.** \$2200
    ii. \$5.50
    iii. \$3.00; \$3.00
    b. **i.** \$24 640
    ii. \$61.60
    iii. \$43.20; \$43.21
    c. **i.** \$5020
    ii. \$12.55
    iii. \$0.03; \$0.03
    d. **i.** \$2705
    ii. \$6.76
    iii. \$4.99; \$4.99
11. 2000
12. **a.** moving away from its starting point
    b. moving away from the origin and towards its starting position
13. **a.** $t = \frac{2}{3}$
    b. Yes, since $a > 0$ for all $t > 0$, the particle is accelerating
14. 27.14 cm by 27.14 cm for the base and height 13.57 cm
15. length 190 m, width approximately 63 m
16. 31.6 m by 11.6 m by 4.2 m
17. radius 4.3 cm, height 8.6 cm
18. Run the pipe 7.2 km along the river shore and then cross diagonally to the refinery.
19. 10:35 p.m.
20. \$204 or \$206
21. The pipeline meets the shore at a point *C*, 5.7 km from point *A*, directly across from *P*.
22. 11.35 cm by 17.02 cm
23. The two identical brick sides should have length 25 m; the fenced side and the corresponding brick side should have length 40 m.
24. 2:23 p.m.
25. 3.2 km from point *C*
26. **a.** absolute maximum: $f(7) = 41$,
    absolute minimum: $f(1) = 5$
    b. absolute maximum: $f(3) = 36$,
    absolute minimum: $f(-3) = -18$
    c. absolute maximum: $f(5) = 67$,
    absolute minimum: $f(-5) = -63$
    d. absolute maximum: $f(4) = 2752$,
    absolute minimum: $f(-2) = -56$
27. **a.** 62.9 m **c.** 3.6 m/s^2
    b. 4.7 s
28. **a.** $f''(2) = 60$ **d.** $f''(1) = -\frac{5}{16}$
    b. $f''(-1) = 26$ **e.** $f''(4) = -\frac{1}{108}$
    c. $f''(0) = 192$ **f.** $f''(8) = -\frac{1}{72}$
29. **a.** position: 1, velocity: $\frac{1}{6}$,
    acceleration: $-\left(\frac{1}{18}\right)$, speed: $\frac{1}{6}$
    b. position: $\frac{8}{3}$, velocity: $\frac{4}{9}$,
    acceleration: $\frac{10}{27}$, speed: $\frac{4}{9}$
30. **a.** $v(t) = \frac{2}{3}(t^2 + t)^{-\frac{1}{3}}(2t + 1)$,
    $a(t) = \frac{2}{9}(t^2 + t)^{-\frac{4}{3}}(2t^2 + 2t - 1)$
    b. 1.931 m/s
    c. 2.36 m/s
    d. undefined
    e. 0.141 m/s^2

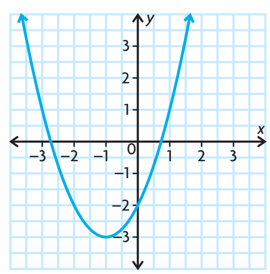
Chapter 3 Test, p. 160

1. **a.** $y'' = 14$
   b. $f''(x) = -180x^3 - 24x$
   c. $y'' = 60x^{-5} + 60x$
   d. $f''(x) = 96(4x - 8)$
2. **a.** $v(3) = -57$,
   $a(3) = -44$
   b. $v(2) = 6$,
   $a(2) = -24$
3. **a.** $v(t) = 2t - 3$,
   $a(t) = 2$
   b. -0.25 m
   c. 1 m/s
   d. between $t = 0$ s and $t = 1.5$ s
   e. 2 m/s
4. **a.** min: -63, max: 67
   b. min: 6, max: 10
5. **a.** 2.1 s
   b. about 22.9 m
6. 250 m by 166.7 m
7. 162 mm by 324 mm by 190 mm
8. \$850/month

Chapter 4

Review of Prerequisite Skills, pp. 162–163

1. **a.** $y = -\frac{3}{2}$ or $y = 1$
   b. $x = 7$ or $x = -2$
   c. $x = -\frac{5}{2}$
   d. $y = 1$ or $y = -3$ or $y = -2$
2. **a.** $x < -\frac{7}{3}$
   b. $x \le 2$
   c. $-1 < t < 3$
   d. $x < -4$ or $x > 1$
3. **a.**



b.